Charge- and surface potential-based MOSFET models,

what are the differences?

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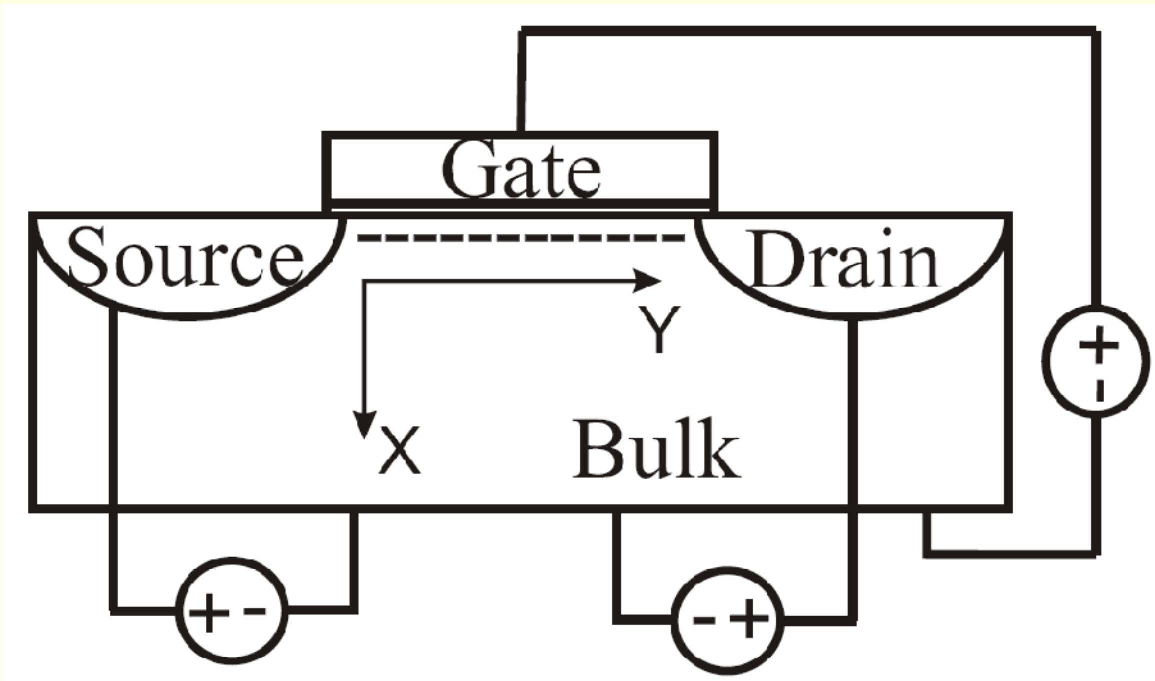
The “golden” reference: Pao-Sah model

2-D problem separated into two 1-D problems:



Vertical 1-D field electrostatics control conduction charge

Input equation

Longitudinal 1-D field controls current flow

Output equation

The Pao-Sah model-2

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

μ: carrier mobility
W : channel width
Q'_I : inversion charge density
$V_S \leq V_C \leq V_D$,
y: distance along the channel

⇓

$$I_D = \frac{W}{L}\int_{V_S}^{V_D} \mu\left(-Q_I'\right)dV_C$$

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G.V_S} = -\frac{W}{L}\mu Q_I'(V_D, V_G)$$

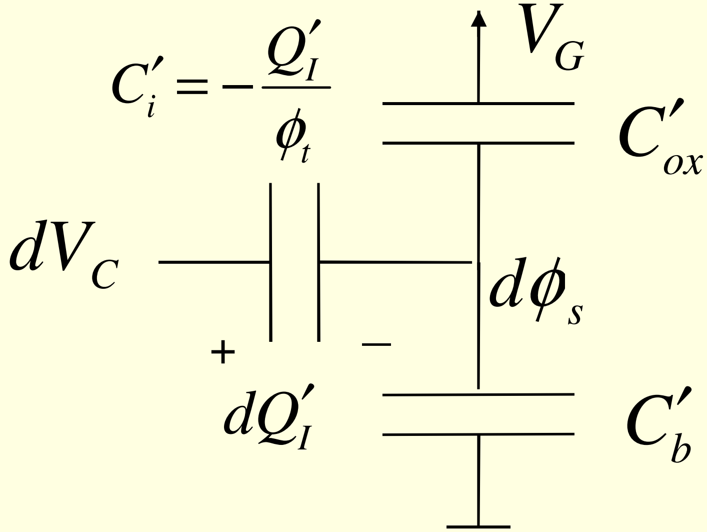
⇑

valid from weak inversion to strong inversion

Charge sheet model 1: fundamentals

Minority carriers are at the interface Si-SiO_2 where $\phi = \phi_s$

$$Q_I' \propto e^{\phi_s/\phi_t} \quad \Rightarrow \quad C_i' = -\frac{dQ_I'}{d\phi_s} = -\frac{Q_I'}{\phi_t}$$



$$dQ_I' = C_i'\left(dV_C - d\phi_s\right) \quad \Rightarrow \quad dV_C = d\phi_s - \phi_t \frac{dQ_I'}{Q_I'}$$

Charge sheet model 2: drain current

Pao-Sah model

$$I_D = -\mu W Q_I' \frac{dV_C}{dy}$$

Charge sheet approximation

$$dV_C = d\phi_s - \phi_t \frac{dQ_I'}{Q_I'}$$

$$I_D = I_{drift} + I_{diff} = -\mu W Q_I' \frac{d\phi_s}{dy} + \mu W \phi_t \frac{dQ_I'}{dy}$$

Charge sheet model 3: Brews' model

$$I_D = I_{drift} + I_{diff} =$$

$$-\mu W Q_I' \frac{d\phi_s}{dy} + \mu W \phi_t \frac{dQ_I'}{dy} \qquad Q_I' = -C_{ox}'\left(V_G - V_{FB} - \phi_s - \gamma\sqrt{\phi_s - \phi_t}\right)$$

$$I_{drift} = \mu \frac{W}{L} C'_{ox} \left\{ (V_G - V_{FB})(\phi_{sL} - \phi_{s0}) - \frac{1}{2}\left(\phi_{sL}^{\ 2} - \phi_{s0}^{\ 2}\right) - \frac{2}{3}\gamma\left[(\phi_{sL} - \phi_t)^{3/2} - (\phi_{s0} - \phi_t)^{3/2}\right]\right\}$$

$$I_{diff} = \mu \frac{W}{L} C'_{ox}\, \phi_t \left\{ (\phi_{sL} - \phi_{s0}) + \gamma\left[(\phi_{sL} - \phi_t)^{1/2} - (\phi_{s0} - \phi_t)^{1/2}\right]\right\}$$

Simplified charge sheet (SCS) models : depletion charge variation along the channel is linearized

$$Q_I' = -C_{ox}'\left(V_G - V_{FB} - \phi_s - \gamma\sqrt{\phi_s - \phi_t}\right)$$

At constant V_G

$$dQ_I' = (C_{ox}' - \frac{dQ_B'}{d\phi_s})d\phi_s = (C_{ox}' + C_b')d\phi_s = nC_{ox}'d\phi_s$$

Assuming a constant depletion capacitance along the channel the 1/2 and 3/2 power terms in Brews' formula are avoided

SCS models: "charge"-based model

$$I_D = -\mu_n W Q_I' \frac{d\phi_s}{dy} + \mu_n W \phi_t \frac{dQ_I'}{dy}$$

$$dQ_I' = (C_{ox}' + C_b')d\phi_s = nC_{ox}'d\phi_s$$

$$I_D = -\frac{\mu_n W}{nC_{ox}'}(Q_I' - \phi_t nC_{ox}')\frac{dQ_I'}{dy}$$

Integrating along the channel yields

$$I_D = \frac{\mu_n W}{L}\left[\frac{Q_{IS}'^{\,2} - Q_{ID}'^{\,2}}{2nC_{ox}'} - \phi_t\left(Q_{IS}' - Q_{ID}'\right)\right]$$

SCS models: "surface potential"-based model

$$I_D = \frac{\mu W}{L}\left(\frac{Q'_{IS}+Q'_{ID}}{2} - nC'_{ox}\phi_t\right)\left(\frac{Q'_{IS}-Q'_{ID}}{nC'_{ox}}\right)$$

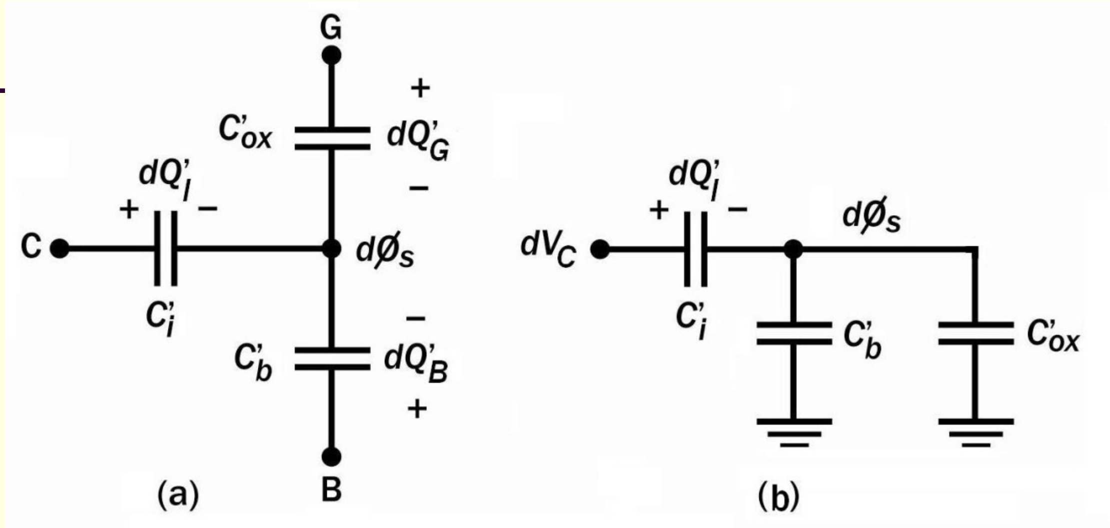
Making the substitutions below

$$nC'_{ox} \leftrightarrow -\alpha \qquad \frac{Q'_{IS}+Q'_{ID}}{2} \leftrightarrow Q'_{\mathrm{Im}} \qquad Q'_{IS}-Q'_{ID} \leftrightarrow -nC'_{ox}(\phi_{sL}-\phi_{s0})$$

we obtain PSP core expression of the drain current

$$I_D = -\mu \frac{W}{L}(Q'_{\mathrm{Im}} + \alpha\phi_t)(\phi_{sL}-\phi_{s0})$$

The "charge"-based model input equation



$$n = n(V_{GB})$$

$$dQ'_I \left(\frac{1}{C'_{ox} + C'_b} + \frac{1}{C'_i} \right) = dV_C \quad \Rightarrow \quad dQ'_I \left(\frac{1}{nC'_{ox}} - \frac{\phi_t}{Q'_I} \right) = dV_C$$

Integrating from V_C to V_P yields the unified charge control model (UCCM)

$$V_P - V_C = \phi_t \left[\frac{Q'_{IP} - Q'_I}{nC'_{ox}\phi_t} + \ln\left(\frac{Q'_I}{Q'_{IP}} \right) \right]$$

Modeling the bulk charge from accumulation to inversion

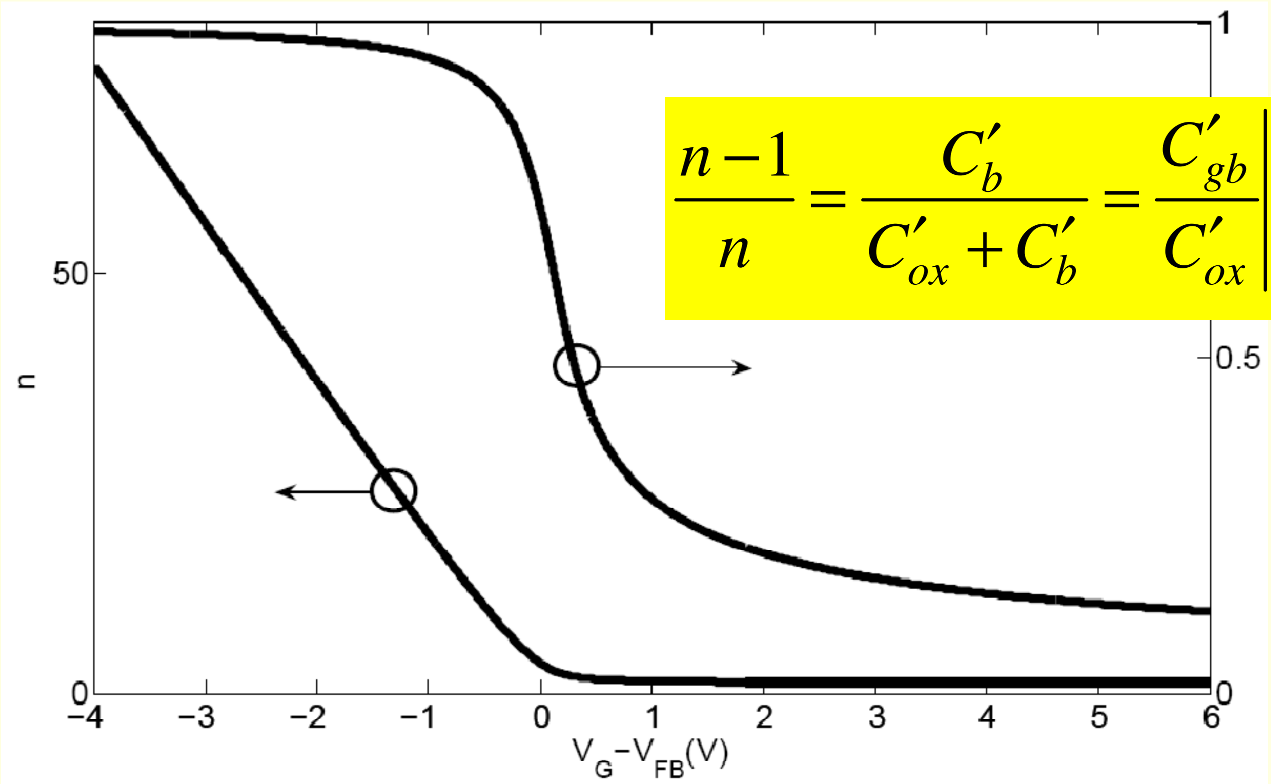
$$Q'_B = -\operatorname{sgn}(\phi_s) C'_{ox} \gamma \sqrt{\phi_s + \phi_t (e^{-\phi_s/\phi_t} - 1)}$$

Defining $\phi_{sa} = \phi_s \big|_{Q'_I = 0}$ it follows that

$$\left(V_G - V_{FB} - \phi_{sa}\right)^2 = \gamma^2 \left[\phi_{sa} + \phi_t \left(e^{-\phi_{sa}/\phi_t} - 1\right)\right]$$

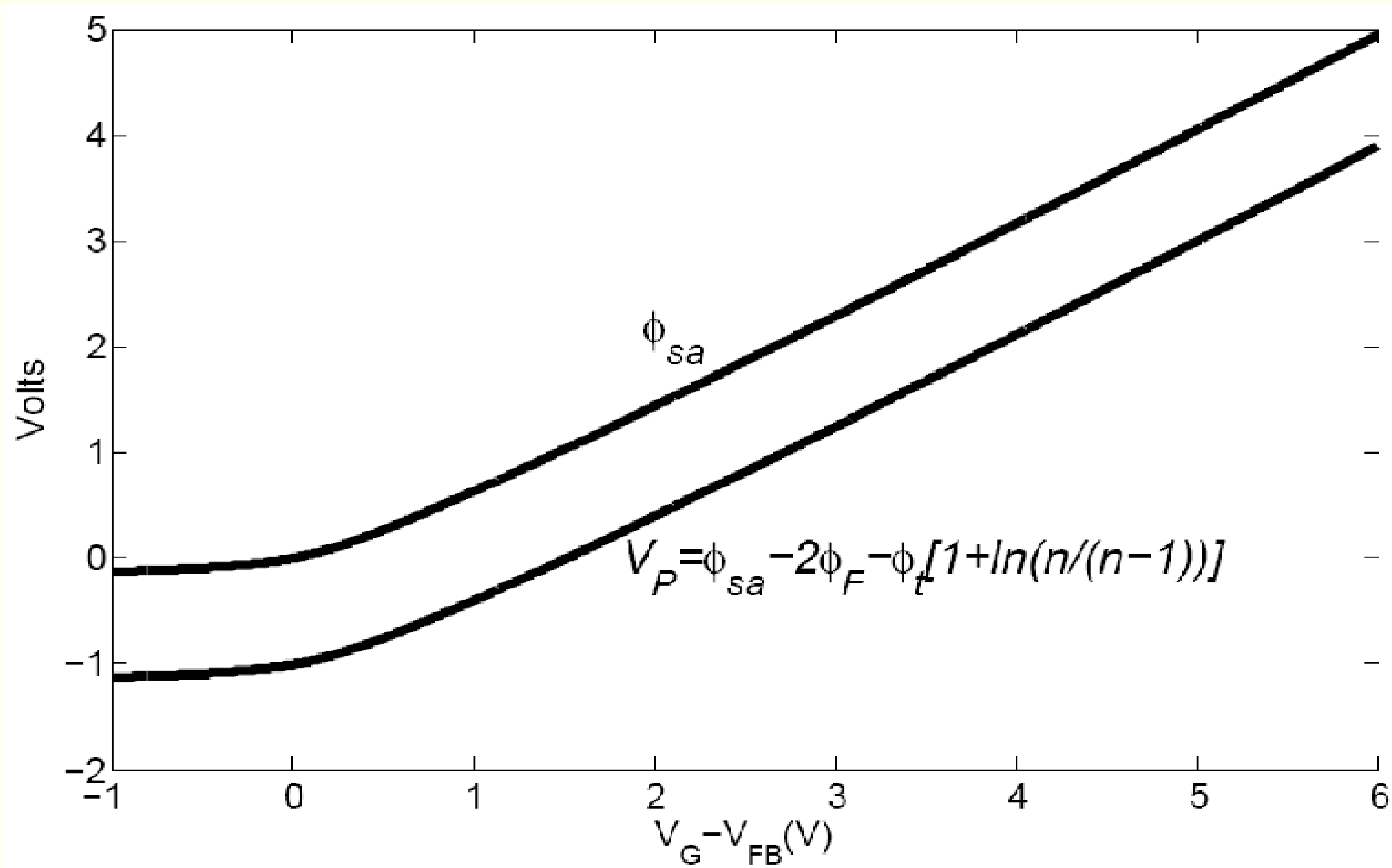
$$n = 1 + \frac{C'_b}{C'_{ox}} = 1 + \frac{\gamma\left(1 - e^{-\phi_{sa}/\phi_t}\right)}{2\operatorname{sgn}\left(\phi_{sa}\right)\sqrt{\phi_{sa} + \phi_t\left(e^{-\phi_{sa}/\phi_t} - 1\right)}}$$

Slope factor and ratio (n-1)/n from accumulation to inversion



$$\frac{n-1}{n} = \frac{C'_b}{C'_{ox} + C'_b} = \left.\frac{C'_{gb}}{C'_{ox}}\right|_{Q'_I=0}$$

Modeling from Accumulation to Inversion: Surface Potential and Pinch-off Voltage (V_P)



The "surface potential"-based model input equation

- Exact calculation of the surface potential

$$\left(V_G - V_{FB} - \phi_t\right)^2 = \gamma^2 [\phi_t \left(e^{-\phi_s/\phi_t} - 1\right)$$

$$+ \phi_s + \phi_t e^{-(2\phi_F + V_C)/\phi_t} \left(e^{\phi_s/\phi_t} - 1\right)]$$

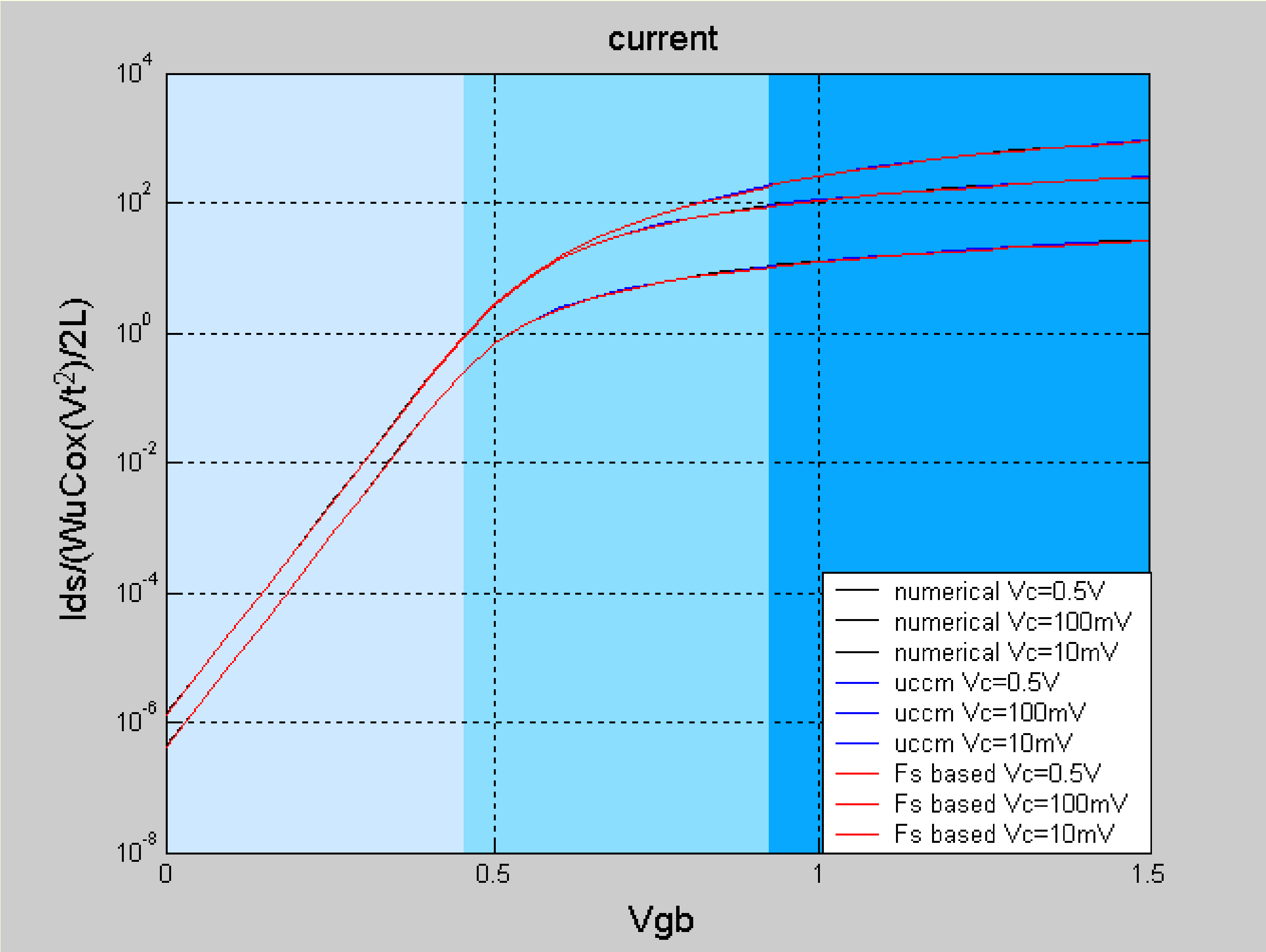
⇩

$$Q'_B = -\operatorname{sgn}(\phi_s) C'_{ox} \gamma \sqrt{\phi_s + \phi_t (e^{-\phi_s/\phi_t} - 1)}$$

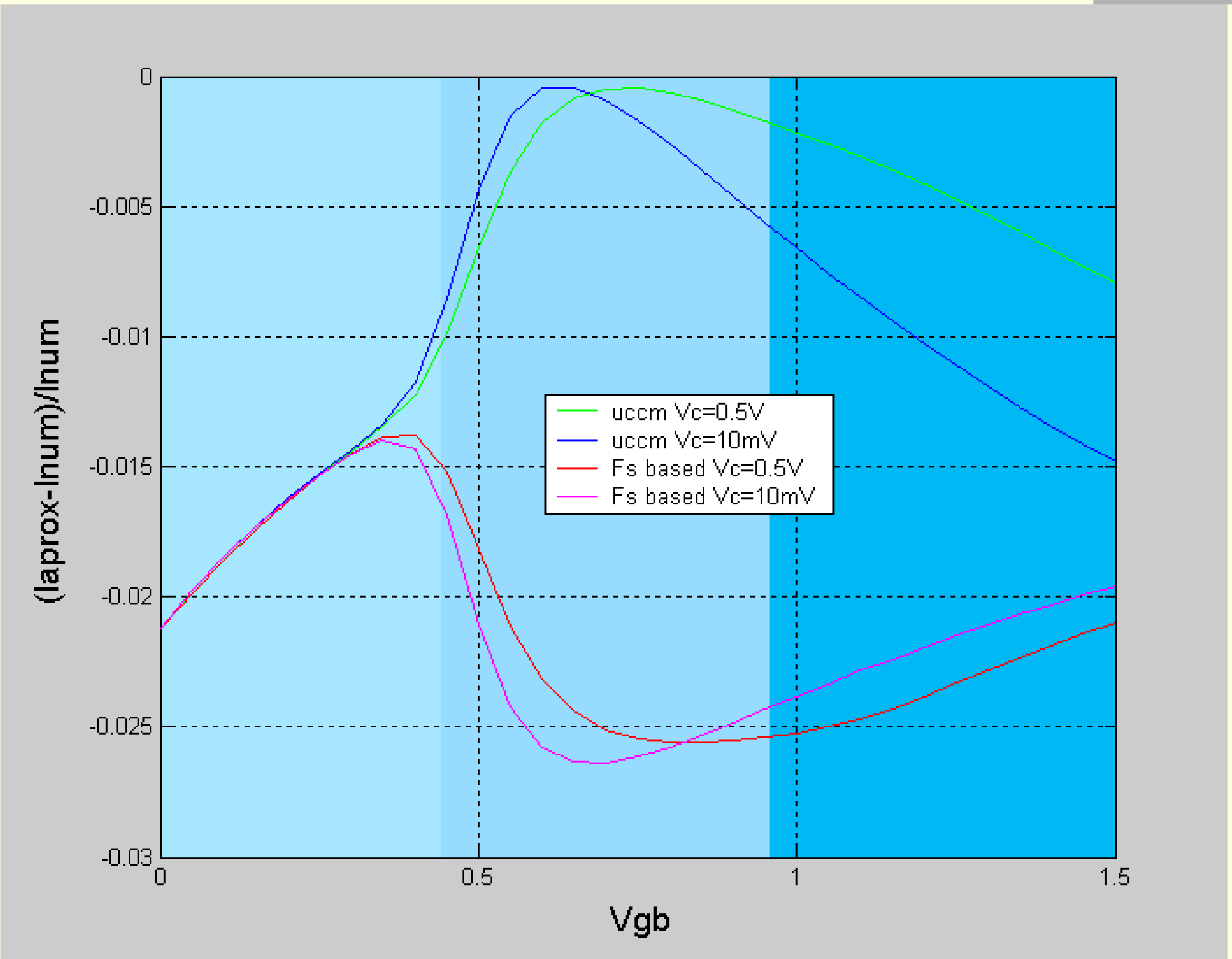
⇩

$$Q'_I = -C'_{ox} \left(V_G - V_{FB} - \phi_s\right) - Q'_B$$

Numerical comparison between SCS and Pao-Sah models 1



Numerical comparison between SCS and Pao-Sah models 2



Consistency of SCS and Pao-Sah models

$$I_D = \frac{W}{L}\int_{V_S}^{V_D} \mu\left(-Q'_I\right) dV_C \qquad I_D = \frac{\mu W}{L}\left[\frac{Q'^{\,2}_{IS} - Q'^{\,2}_{ID}}{2nC'_{ox}} - \phi_t\left(Q'_{IS} - Q'_{ID}\right)\right]$$

$$g_{dPao-Sah} = -\frac{W}{L}\mu Q'_I(V_D, V_G) \qquad g_{dSCS} = \frac{\mu W}{L}\left[\frac{-Q'_{ID}}{nC'_{ox}} + \phi_t\right]\frac{dQ'_{ID}}{dV_D}$$

$$g_{dSCS} = g_{dPao-Sah} \;\Rightarrow\; dQ'_{ID}\left(\frac{1}{nC'_{ox}} - \frac{\phi_t}{Q'_{ID}}\right) = dV_D$$

UCCM!

To maintain consistency between SCS and Pao-Sah models we must calculate the inversion charge density using UCCM!

Calculation of the charge derivatives in the "charge"-based model

$$dQ_I'\left(\frac{1}{nC_{ox}'}-\frac{\phi_t}{Q_I'}\right)=dV_C$$

Unified Charge Control Model (UCCM)

$$\frac{\partial Q_{IS(D)}'}{\partial V_{S(D)}}=nC_{ox}'\frac{Q_{IS(D)}'}{Q_{IS(D)}'-nC_{ox}'\phi_t}$$

$$\frac{\partial Q_I'}{\partial V_G}=-\frac{1}{n}\frac{\partial Q_I'}{\partial V_C}$$

$$\frac{\partial Q_I'}{\partial V_B}=-\frac{n-1}{n}\frac{\partial Q_I'}{\partial V_C}$$

UCCM allows simple and physics based analytical expressions for the capacitive coefficients

Capacitive coefficients for the MOSFET calculated using the UCCM

$$C_{gs} = \frac{2}{3} C_{ox} \frac{1+2\alpha}{(1+\alpha)^2} \frac{q'_{IS}}{1+q'_{IS}} \qquad C_{gd} = \frac{2}{3} C_{ox} \frac{\alpha^2+2\alpha}{(1+\alpha)^2} \frac{q'_{ID}}{1+q'_{ID}}$$

$$C_{gb} = C_{gb} = \frac{n-1}{n} (C_{ox} - C_{gs} - C_{gd}) \quad C_{sd} = -\frac{4}{15} n C_{ox} \frac{\alpha + 3\alpha^2 + \alpha^3}{(1+\alpha)^3} \frac{q'_{ID}}{1+q'_{ID}}$$

…………..etc.

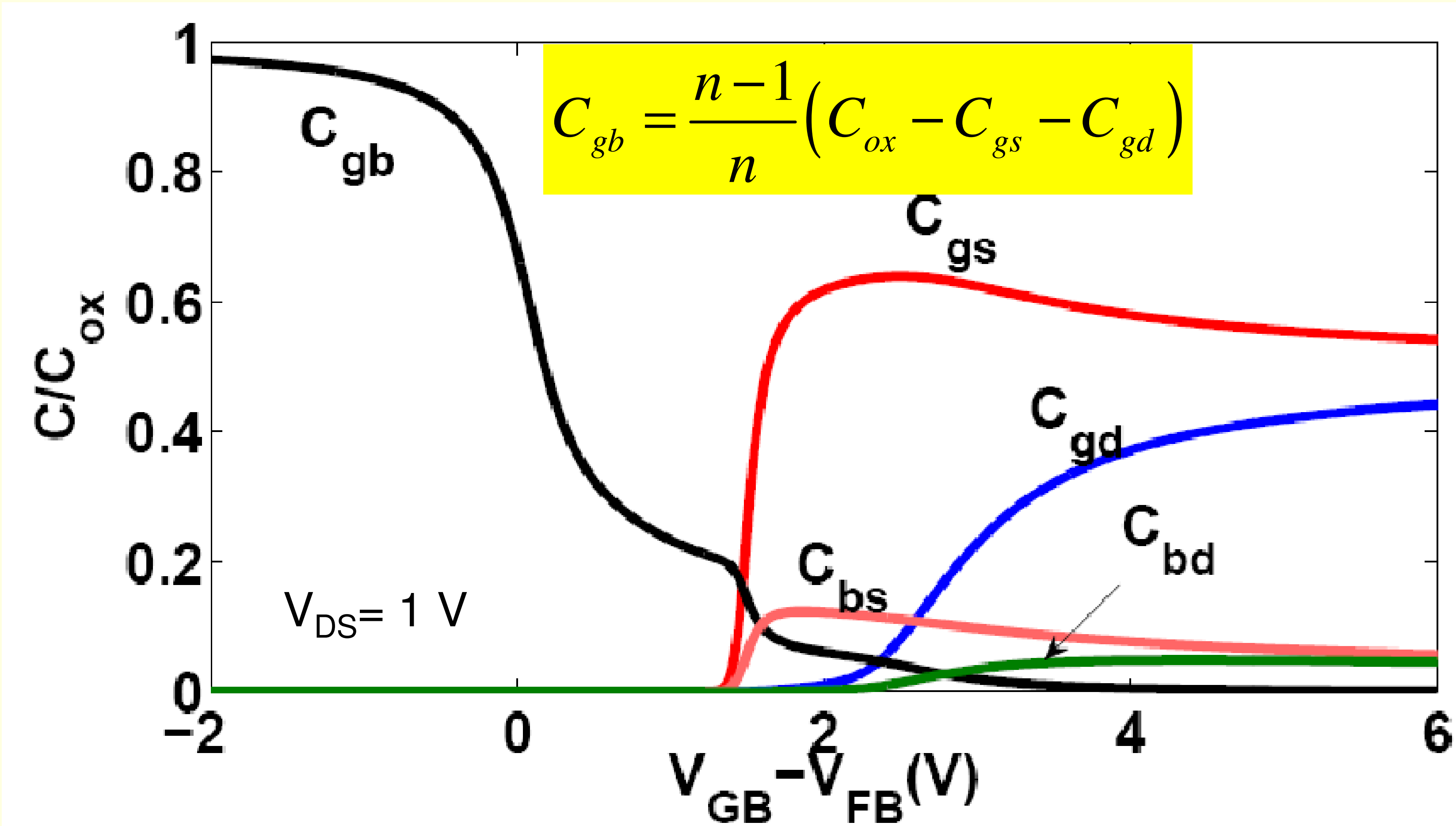
where

$$\alpha = \frac{Q'_{ID} - nC'_{ox}\phi_t}{Q'_{IS} - nC'_{ox}\phi_t}$$

and

$$q'_{IS(D)} = \frac{Q'_{IS(D)}}{-nC'_{ox}\phi_t}$$

The five capacitances of the quasi-static MOSFET model



Some Capacitive coefficients in PSP for V_{DS}= 0 (Xin Li *et al.* , IEEE TED, vol. 56, no. 2, p. 247, Feb. 2009.

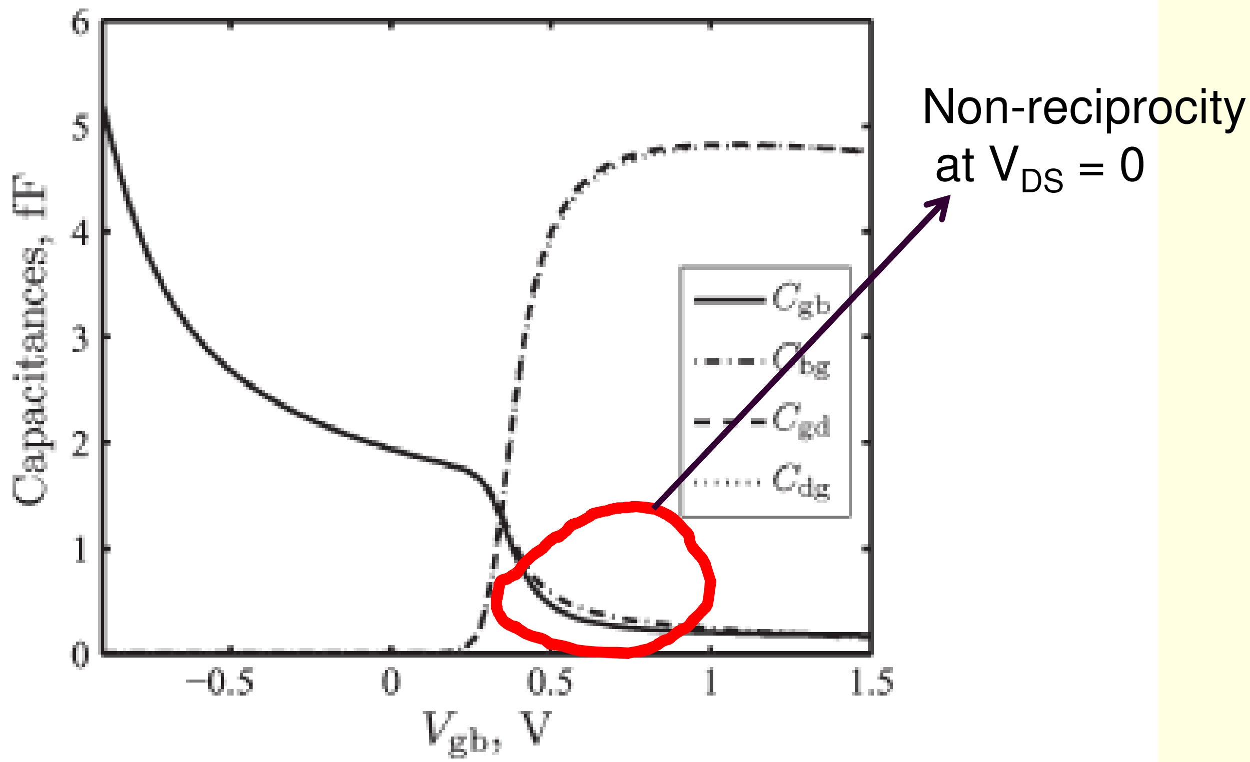
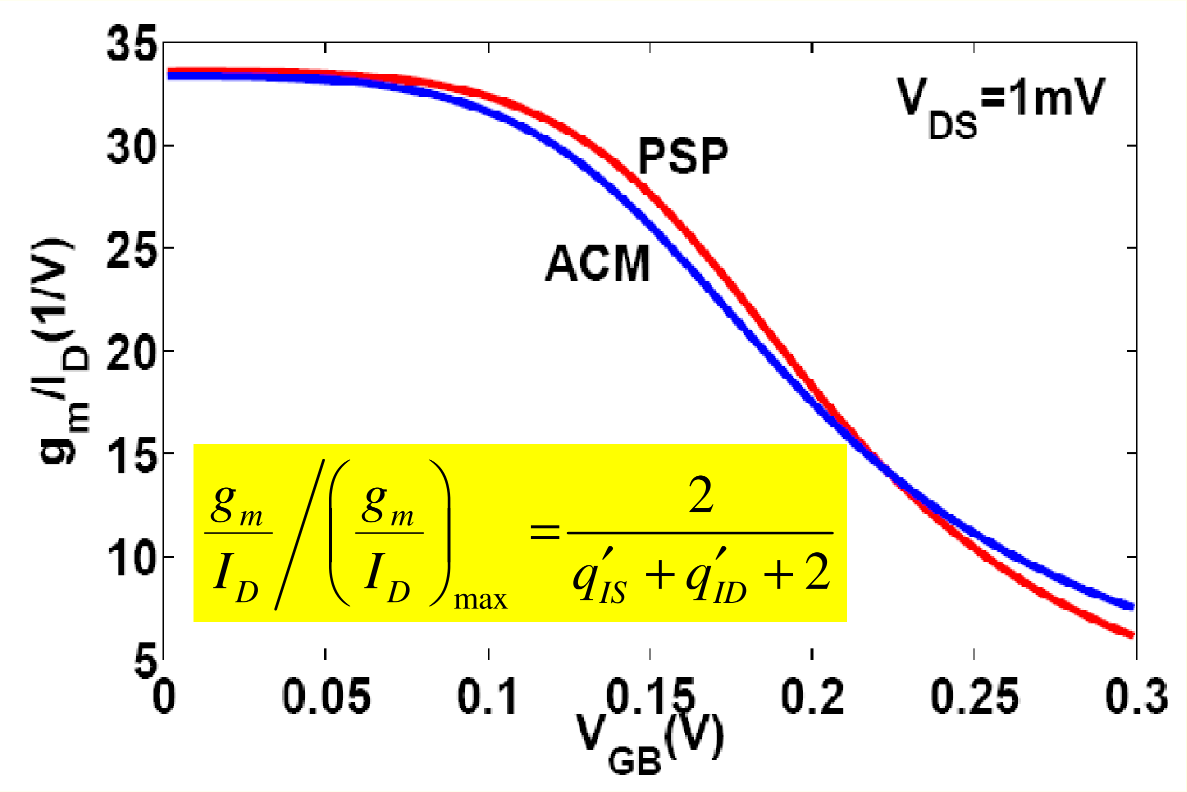


Fig. 12. Transcapacitances C_{gd}, C_{dg}, C_{gb}, and C_{bg} for PSP with $W/L =$ 10/0.08 μm at $V_{ds} = 0$. Default parameters are used.

Parameter extraction



$$\left(g_m / I_D\right)_{max} = 1/\left(n\phi_t\right)$$

$$q'_{IS(D)} = \frac{Q'_{IS(D)}}{Q'_{IP}} = \frac{Q'_{IS(D)}}{-nC'_{ox}\phi_t}$$

Conclusions

- **Charge (Q) - and surface potential (Φ_s)-based MOSFET models are simplified charge sheet models based on the same basic hypotheses**
- **Q and Φ_s –based models differ in the choice of the linearization point and in the resolution of the input electrostatic equation.**
- **Q and Φ_s –based models give similar numerical results in all the operating regions**
- **The Q-based model is fully consistent with the Pao-Sah formula since it uses the same approximations for the input (electrostatic) and output (transport) equations**

Main references

- **C. Galup-Montoro and M. C. Schneider, *MOSFET Modeling For Circuit Analysis And Design*, World Scientific, 2007**

- **J. Watts, C. McAndrew, C. Enz, C. Galup-Montoro, G. Gildenblat, C. Hu, R. van Langevelde, M. Miura-Mattausch, R. Rios and C. Sah, "Advanced compact models for MOSFETs", Workshop on Compact Modeling, Anaheim, USA, May 2005.**

- **C. Galup-Montoro, M. C. Schneider, V. C. Pahim and R. Rios, "Comparison of surface-potential-based and charge-based MOSFET core models" Workshop on Compact Modeling, Anaheim, May 2005.**

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