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Derivation of the unified charge control model and parameter extraction procedure

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Abstract

This paper presents a physical derivation of the unified charge control model (UCCM) [Park C-K, Lee C-Y, Lee K, Moon B-J, Byun YH, Shur M. IEEE Trans Electron Dev 1991;ED-38:399; Lee K, Shur M, Fjeldly TA, Ytterdal T. Semiconductor device modeling for VLSI. Englewoods Cliffs: Prentice Hall, 1993.], which describes the relationship between inversion charge density and the applied voltages in the MOS transistor for any operating region. Using the UCCM and the MOSFET charge model presented in Cunha et al. [Cunha AIA, Schneider MC, Galup-Montoro C. Solid-St Electron 1995;38:1945.], we develop a MOSFET model formulated in terms of the drain current in saturation. Based on this model, a methodology for the extraction of the UCCM parameters is proposed.
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1. NOTATION

C'_{ox} oxide capacitance per unit area (F/m^2)
I_D drain current (A)
I_F forward saturation current (A)
i_f forward normalized current (dimensionless)
I_R reverse saturation current (A)
i_r reverse normalized current (dimensionless)
I_S normalization current (A)
L effective channel length (m)
n slope factor (dimensionless)
Q'_I inversion charge density (C/m^2)
Q'_{ID} inversion charge density at drain (C/m^2)
Q'_{IS} inversion charge density at source (C/m^2)
Q'_{IP} inversion charge density at pinch-off (C/m^2)
t_{ox} oxide thickness (m)
V_C channel voltage = difference between the electron quasi-Fermi potential and the bulk Fermi potential (V)
V_D drain voltage (V)
V_G gate voltage (V)
V_P pinch-off voltage (V)
V_S source voltage (V)
V_{TO} threshold voltage at equilibrium (V)
W effective channel width (m)
ϕ_S surface potential (V)
ϕ_t thermal voltage (V)
γ body effect factor (V$^{1/2}$)
μ carrier mobility (m^2/(V·s))

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2. Introduction

As a consequence of the trend towards shorter channel lengths and reduced supply voltages, MOS devices are expected to often operate in the moderate and weak inversion regions. In this work, we present a formal derivation of UCCM, a MOSFET model applicable to any region of inversion. We first derive a relationship between charge and applied voltages similar to UCCM. Next, we combine the relation between

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charge and voltage and the expression for the drain current derived in [3] to obtain an *I–V* MOSFET model. Finally, we propose a very simple procedure for extracting the parameters of the UCCM model.

3. Derivation of the unified charge control model

From the MOS three terminal charge balance equation [5] (p. 79) it is easy to show [3] that for constant gate-to-bulk potential (V_G)

$$\left.\frac{d\phi_s}{dV_C}\right|_{V_G} = \frac{C'_i}{C'_i + C'_b + C'_{ox}} \tag{1}$$

where ϕ_S is the surface potential, $V_C(x)$ is the difference between the quasi-Fermi potentials of electrons and holes, and C'_i, C'_{ox}, C'_b are the inversion, oxide and depletion capacitances per unit area, respectively.

As shown in [3], for constant gate to substrate potential, the inversion charge density can be approximated by a linear function of the surface potential

$$dQ'_I = (C'_{ox} + C'_b)d\phi_s = nC'_{ox}d\phi_S, \tag{2}$$

where n is the so-called slope factor and is slightly dependent on the gate potential through C'_b.

The substitution of the fundamental approximation of Eq. (2) into Eq. (1) leads to

$$\left.\frac{dQ'_I}{dV_C}\right|_{V_G} = \frac{(C'_{ox} + C'_b)C'_i}{C'_{ox} + C'_b + C'_i}. \tag{3}$$

Eq. (3) is equivalent to the small signal model shown in Fig. 1. In weak, moderate and not very strong inversion, the inversion capacitance can be approximated [3] by

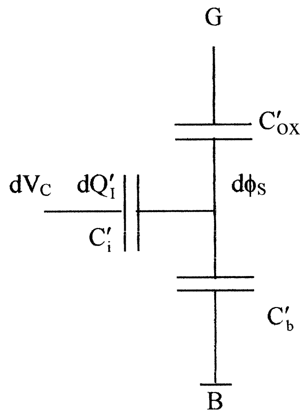


Fig. 1. Small-signal model for the three terminal MOSFET.

$$C'_i = -\frac{Q'_I}{\phi_t}. \tag{4}$$

In very strong inversion, Eq. (4) overestimates the inversion capacitance by a factor of 2, but in this region $C'_i \gg C'_{ox}$, C'_b and, therefore, its exact value is not relevant to calculate the derivative in Eq. (3). Indeed, for very strong inversion, the right-hand side of Eq. (3) can be approximated by $(C'_{ox} + C'_b)$.

The substitution of Eq. (4) into Eq. (3) allows obtaining the following relationship between C'_I and V_C

$$dQ'_I\left(\frac{1}{nC'_{ox}} - \frac{\phi_t}{Q'_I}\right) = dV_C. \tag{5}$$

Integrating Eq. (5) from an arbitrary channel potential V_C to the pinch-off voltage V_P, yields

$$\frac{Q'_{IP} - Q'_I}{nC'_{ox}} + \phi_t \ln\left(\frac{Q'_I}{Q'_{IP}}\right) = V_P - V_C, \tag{6}$$

where Q'_{IP} is the value of Q'_I at pinch-off.

Substituting into Eq. (6) the first order approximation of V_P presented in [4]

$$V_P \cong \frac{V_G - V_{T0}}{n} \tag{7}$$

gives

$$Q'_{IP} - Q'_I + nC'_{ox}\phi_t \ln\left(\frac{Q'_I}{Q'_{IP}}\right) = C'_{ox}(V_G - V_{T0} - nV_C), \tag{8}$$

where V_{T0} is the threshold voltage in equilibrium.

Eq. (8) is almost identical to the unified charge control model [1, 2], which was first presented as an empirical result. In fact, the UCCM is based on the linear dependence of the inversion capacitance on the inversion charge, the charge-sheet approximation and the linear relationship between inversion charge density and surface potential. In Fig. 2 we compare the values of the inversion charge density calculated by either using Eq. (6) or the expression of Q'_I in terms of ϕ_S from the charge-sheet approximation [5] (p. 79), with ϕ_S numerically evaluated in terms of V_C. It can be readily verified that the charge-sheet model and expression (6) are almost equivalent throughout the entire inversion region of operation.

4. Long channel *I–V* modeling

The model presented in [3] consists of a set of expressions for the MOSFET static and dynamic characteristics. These expressions are functions of the inversion charge densities (calculated at source (Q'_{IS}) and drain (Q'_{IS})) and their derivatives with respect to

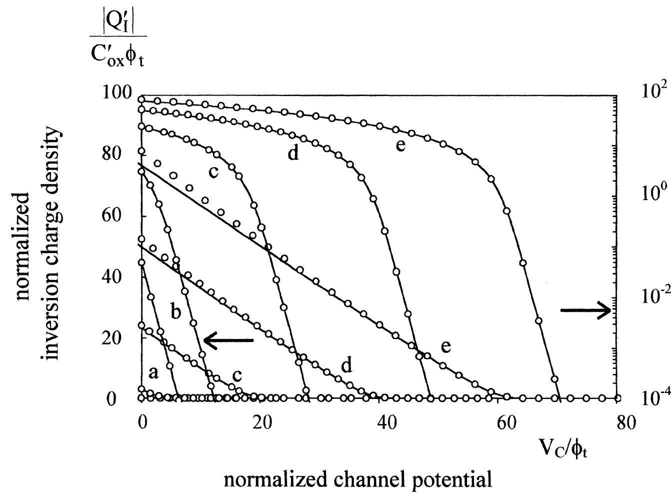


Fig. 2. Normalized inversion charge density vs. normalized channel potential calculated from: (———) expression (6) and (○) the implicit charge-sheet expression [5] (p. 79), with the surface potential numerically evaluated. $V_G - V_{T0}$ equal to: (a) $-5\phi_t$, (b) $5\phi_t$, (c) $30\phi_t$ (d) $60\phi_t$, (e) $90\phi_t$.

the source and drain voltages. Combining the model in [3] with expression (6), allows one to express the MOSFET characteristics in terms of the applied gate-to-bulk (V_G), source-to-bulk (V_S) and drain-to-bulk (V_D) voltages.

In [3], the MOSFET drain current is determined by substituting the basic approximation of Eq. (2) into the expression

$$I_D = \mu W\left(-Q'_I\frac{d\phi_S}{dx} + \phi_t\frac{dQ'_I}{dx}\right) \quad (9)$$

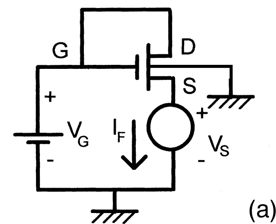
and integrating along the channel length, thus obtaining:

$$I_D = I_F - I_R \quad (10a)$$

$$I_{F(R)} = \mu n C'_{ox}\frac{W}{L}\frac{\phi_t^2}{2}\left[\left(\frac{Q'_{IS(D)}}{nC'_{ox}\phi_t}\right)^2 - 2\frac{Q'_{IS(D)}}{nC'_{ox}\phi_t}\right], \quad (10b)$$

where μ is the electron mobility, W is the channel width and L is the channel length.

The expressions of Eqs. (10a)–(b) describe the drain current of a long-channel MOS transistor as the difference between two symmetric terms: the forward saturation current I_F, which depends only on V_S and V_G, and the reverse saturation current I_R, which depends only on V_D and V_G. In forward saturation, I_R vanishes and the drain current equals I_F. Hence, I_F can be measured from the circuit shown in Fig. 3. Analogously, I_D approaches I_R in reverse saturation.



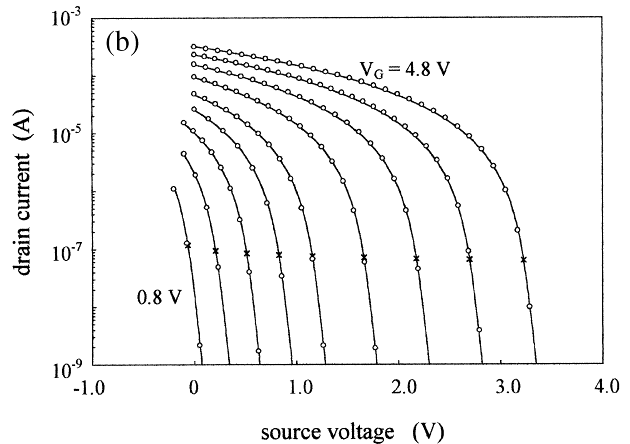


Fig. 3. (a) Circuit configuration for measuring the common-gate characteristics in saturation. (b) Common-gate characteristics in saturation of an NMOS transistor with $t_{ox} = 280$ Å and $W = L = 25$ μm ($V_G = 0.8$, 1.2, 1.6, 2.0, 2.4, 3.0, 3.6, 4.2 and 4.8 V): (———) simulated curves calculated from Eq. (12); (○) measured curves; (*) measured points corresponding to logarithmic derivative equal to $-2/3\phi_t$ and $V_S = V_P$ ($i_f = 3$).

From Eqs. (10a)–(b), we derive the relationship between the inversion charge density calculated at each channel end and its corresponding current component (I_F or I_R)

$$-\frac{Q'_{IS(D)}}{nC'_{ox}\phi_t} = \sqrt{1+i_{f(r)}} - 1, \quad (11a)$$

where $i_{f(r)}$ is the forward (reverse) normalized current [4] defined as

$$i_{f(r)} = \frac{I_{F(R)}}{I_S} \quad (11b)$$

and I_S is the normalization current, given by

$$I_S = \mu n C'_{ox} \frac{W}{L}\frac{\phi_t^2}{2} \quad (11c)$$

which is equal to the threshold current in saturation, I_t, defined in Ref. [2] (p. 311) and is four times smaller than the homonym presented in [4].

The substitution of Eq. (11a) into Eq. (6) leads to the following universal relationship between the forward (reverse) normalized current and the source (drain) normalized voltage

$$\frac{V_P - V_{S(D)}}{\phi_t} = \sqrt{1+i_{f(r)}} - \sqrt{1+i_P} + \ln\left(\frac{\sqrt{1+i_{f(r)}}-1}{\sqrt{1+i_P}-1}\right), \quad (12)$$

where i_P is the value of $i_{f(r)}$ for $V_{S(D)} = V_P$.

Expression of Eq. (12) is independent of technology, transistor dimensions and temperature. It is noticeable from Eq. (12) that the forward and reverse saturation characteristics (I_F vs. V_S or I_R vs. V_D) are exactly the same.

5. Parameter extraction and results

The parameters of the long-channel model can be extracted using the following procedure:

1. To determine the normalization current $I_S(V_G)$, we measure the 'common-gate' characteristics in saturation ($V_D = V_G$) for several values of V_G, as illustrated in Fig. 3. The logarithmic derivative of the forward current calculated from Eq. (12) is given by

$$\frac{\partial \ln I_F}{\partial V_S} = -\frac{2}{\phi_t(1+\sqrt{1+i_f})}. \quad (13)$$

2. This derivative corresponds to the slope of the curves in Fig. 3 and is a function of the forward normalized current. Therefore, a particular value of this derivative can be used to determine I_S. For instance, we can measure the values I_{F0} of the forward current such that the logarithmic derivative is equal to $-2/3\phi_t(i_f = 3)$. These values (I_{F0}) are depicted by the asterisks Fig. 3. Therefore, we can readily calculate $I_S = I_{F0}/3$.
3. The pinch-off voltage $V_P(V_G)$ is also extracted from the common-gate characteristics. Here we assume that the inversion charge density at pinch-off is $Q'_{IP} = -nC'_{ox}\phi_t$, which implies that $i_P = 3$. On the other hand, we adopt for V_P and n the definitions

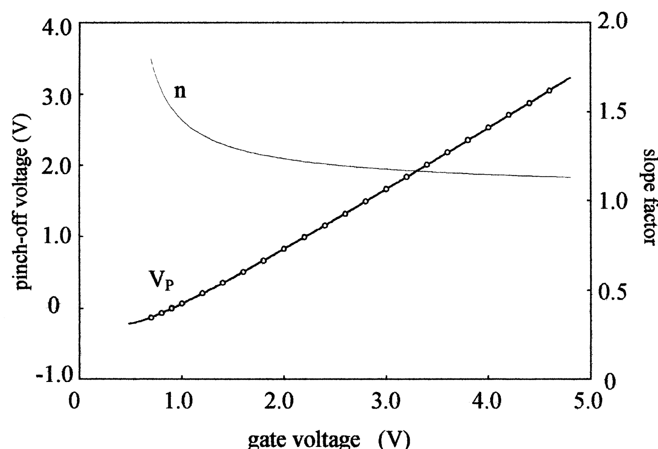


Fig. 4. Pinch-off voltage and slope factor vs. gate voltage for an NMOS transistor with $t_{ox} = 280$ Å and $W = L = 25$ μm: (○) extracted values of V_P; (———) values of V_P and n calculated from theoretical expressions in [4].

in [4]. According to Eq. (12), V_P is the value of V_S such that $I_F = I_S.i_P$. Adopting, $i_P = 3$, V_P is equal to V_S for the asterisks in Fig. 3. Fig. 4 exhibits the extracted and fitted values of the pinch-off voltage, as well as the calculated values of the slope factor. The extraction procedure proposed here is fully independent of the knowledge of transistor dimensions and technological parameters in opposition to the methodology presented in [2].

4. The dependence of the mobility on the electrical transversal field can be extracted from the variations of I_S and n with respect to V_G.

The theoretical and measured 'common-gate characteristic' (I_D vs. V_S for constant V_G) in saturation are shown in Fig. 3 for an NMOS transistor whose oxide thickness (t_{ox}) is 280 Å and whose channel dimensions are $W = L = 25$ μm. The theoretical values have been obtained by using Eq. (12) and agree very well with the experimental results.

6. Conclusions

We have presented the physical approximation underlying the Unified Charge Control Model for MOS transistors. We have verified that the UCCM is fully consistent with the basic approximation of the model derived in [3]. Combining the model presented in [3] with the UCCM, we have derived a MOSFET model whose key variables are the forward and reverse normalized currents. Such a feature is extremely useful, since the bias current has a key role in circuit performance.

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