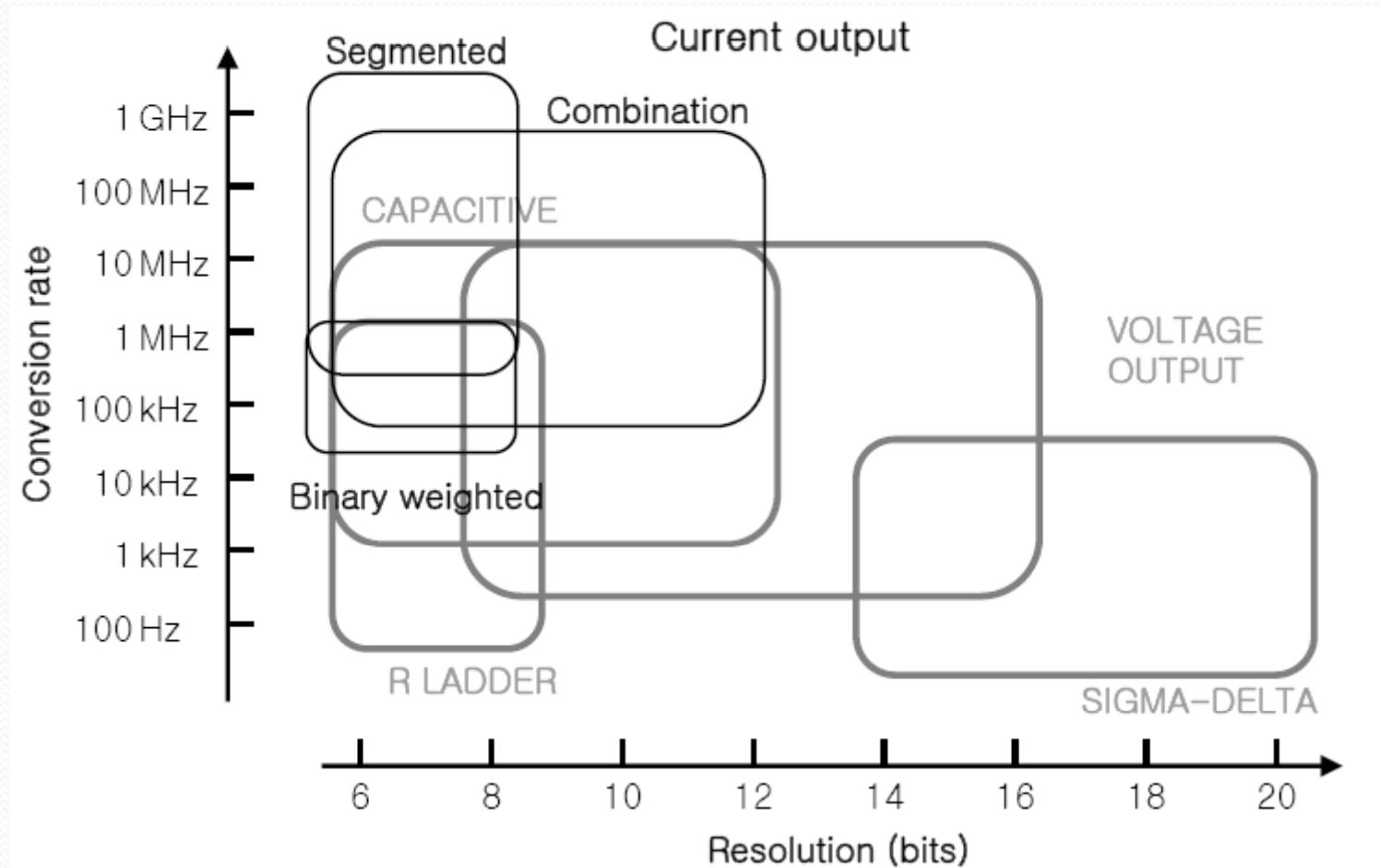
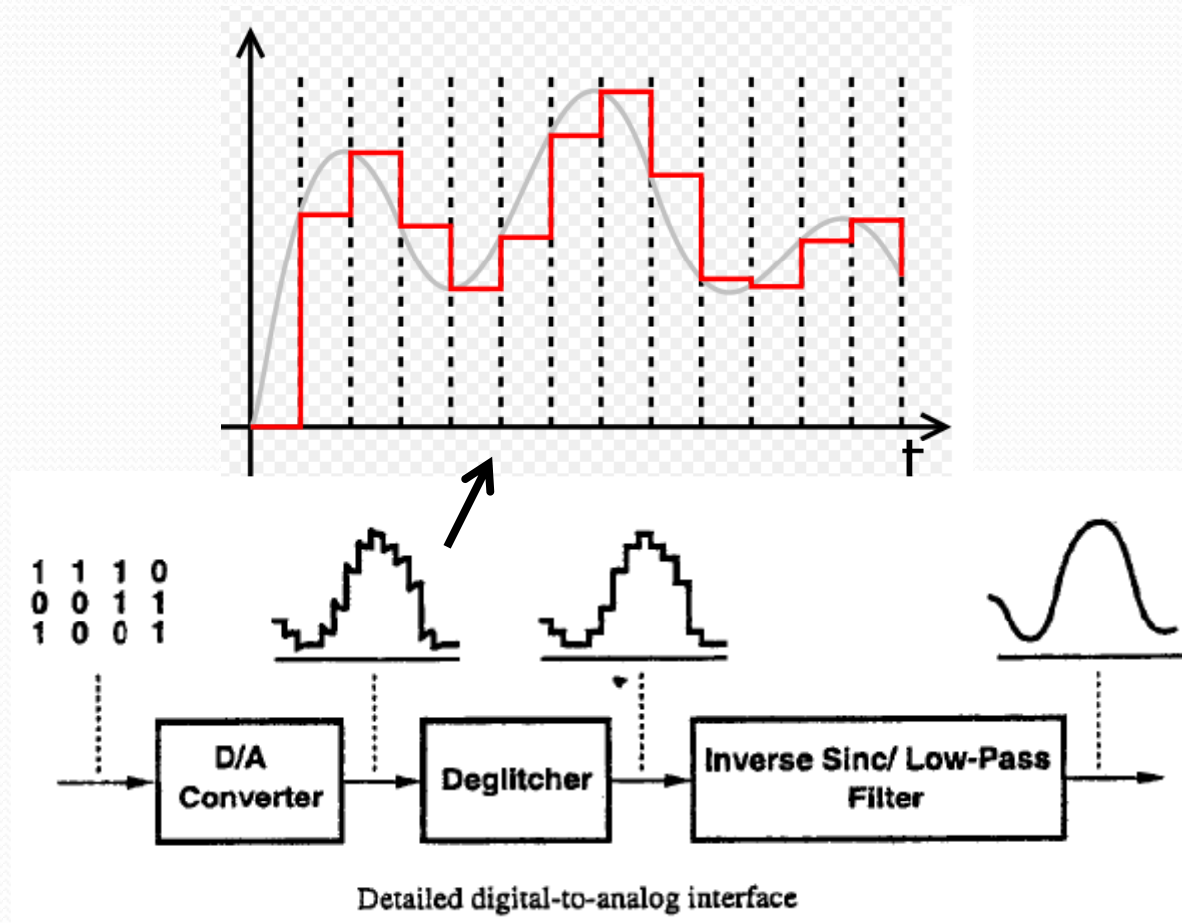
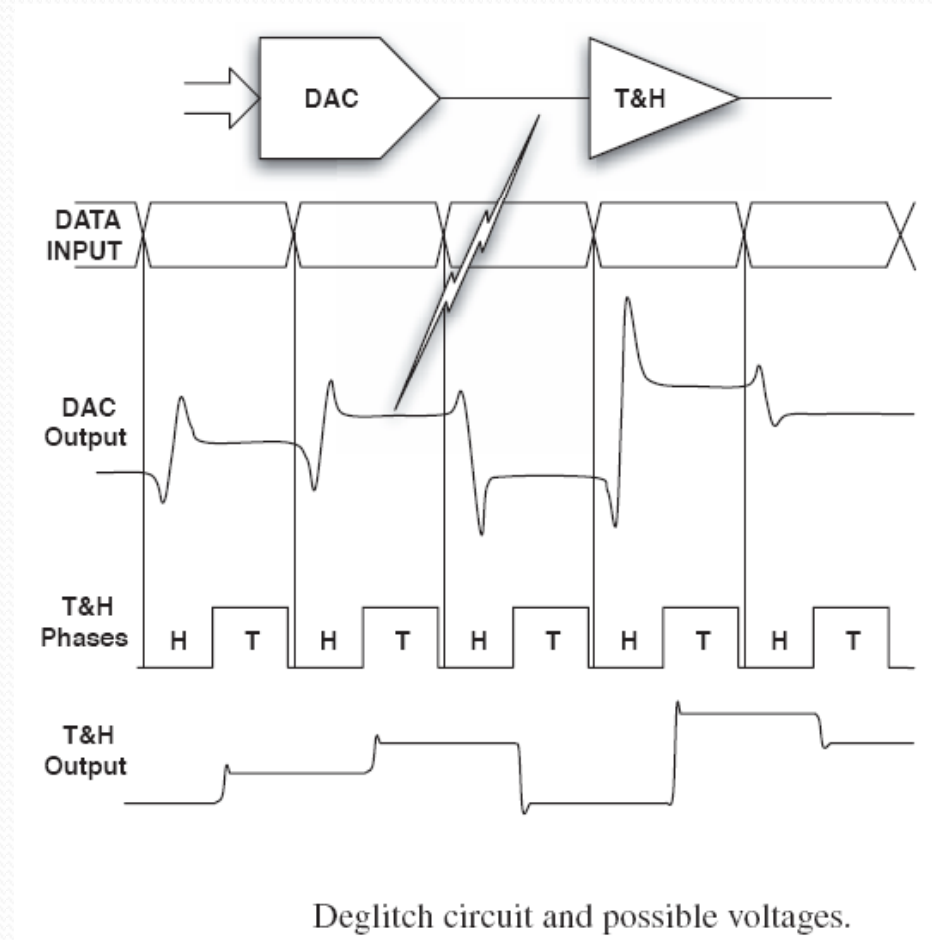


D/A Converters



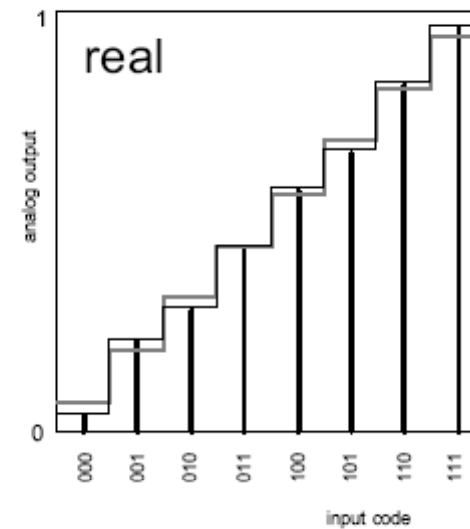
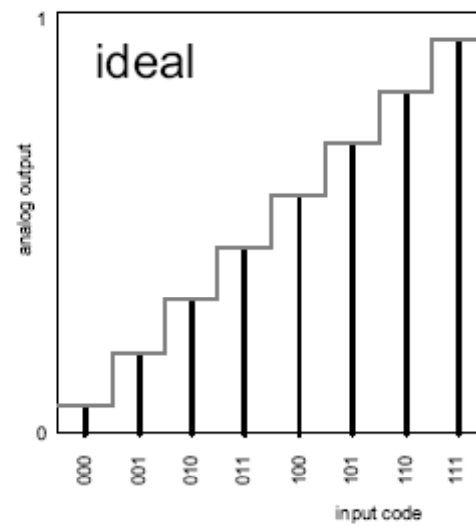


Deglitching



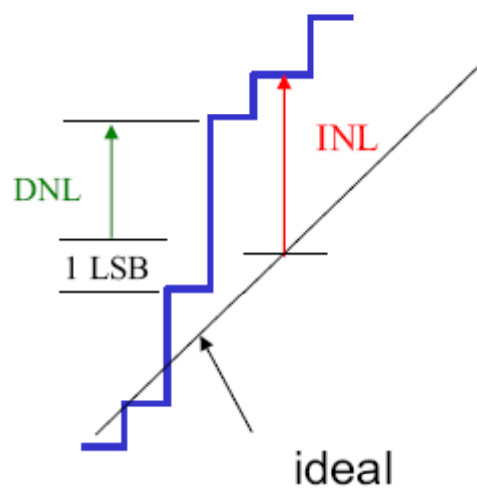
Static transfer characteristic

D to A converters

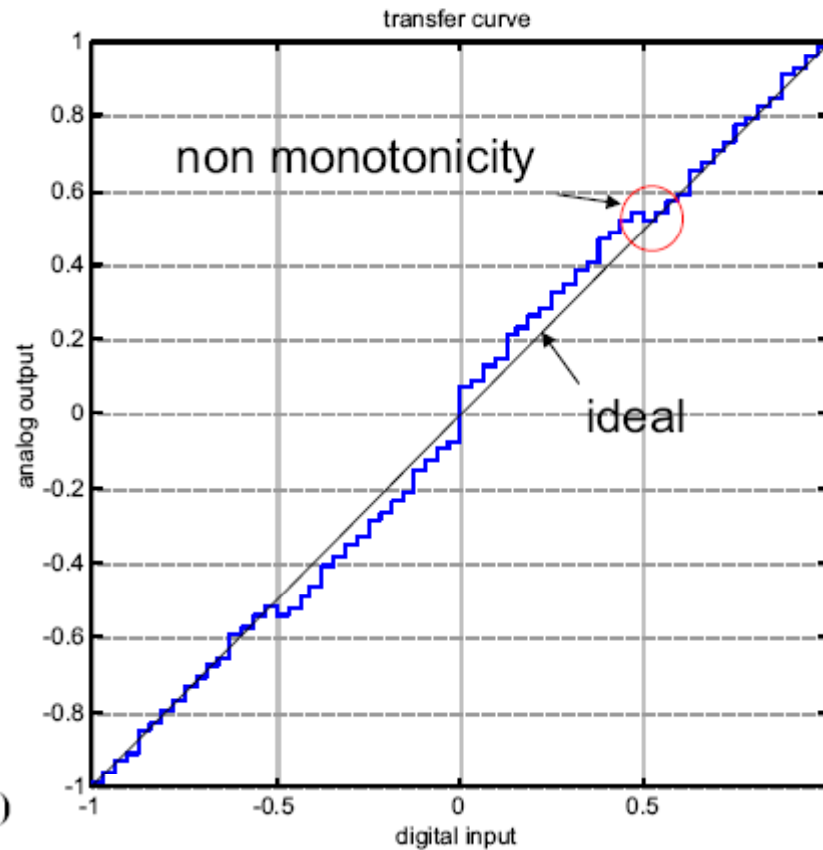


Linearity characterization

- D to A converters

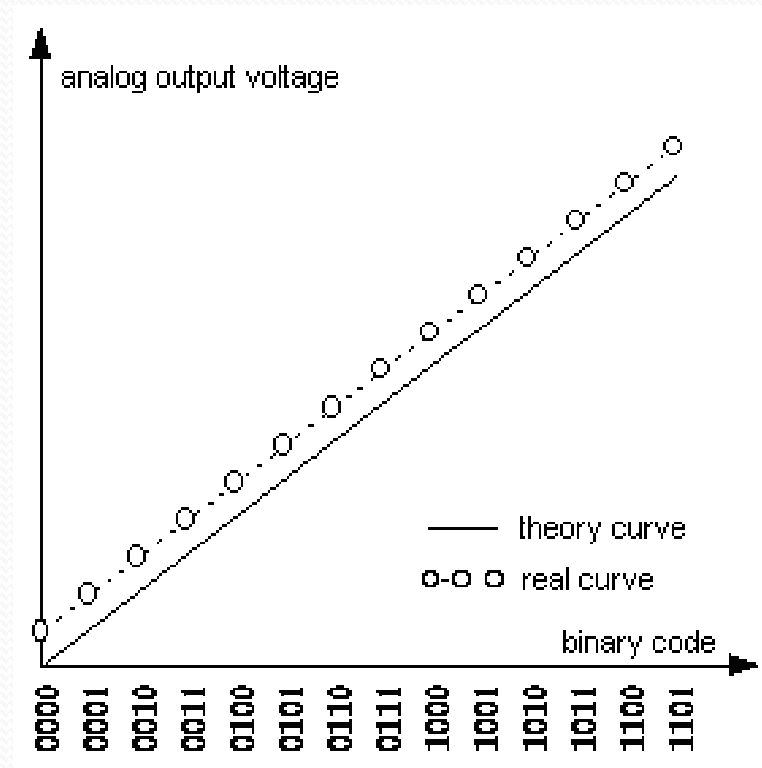


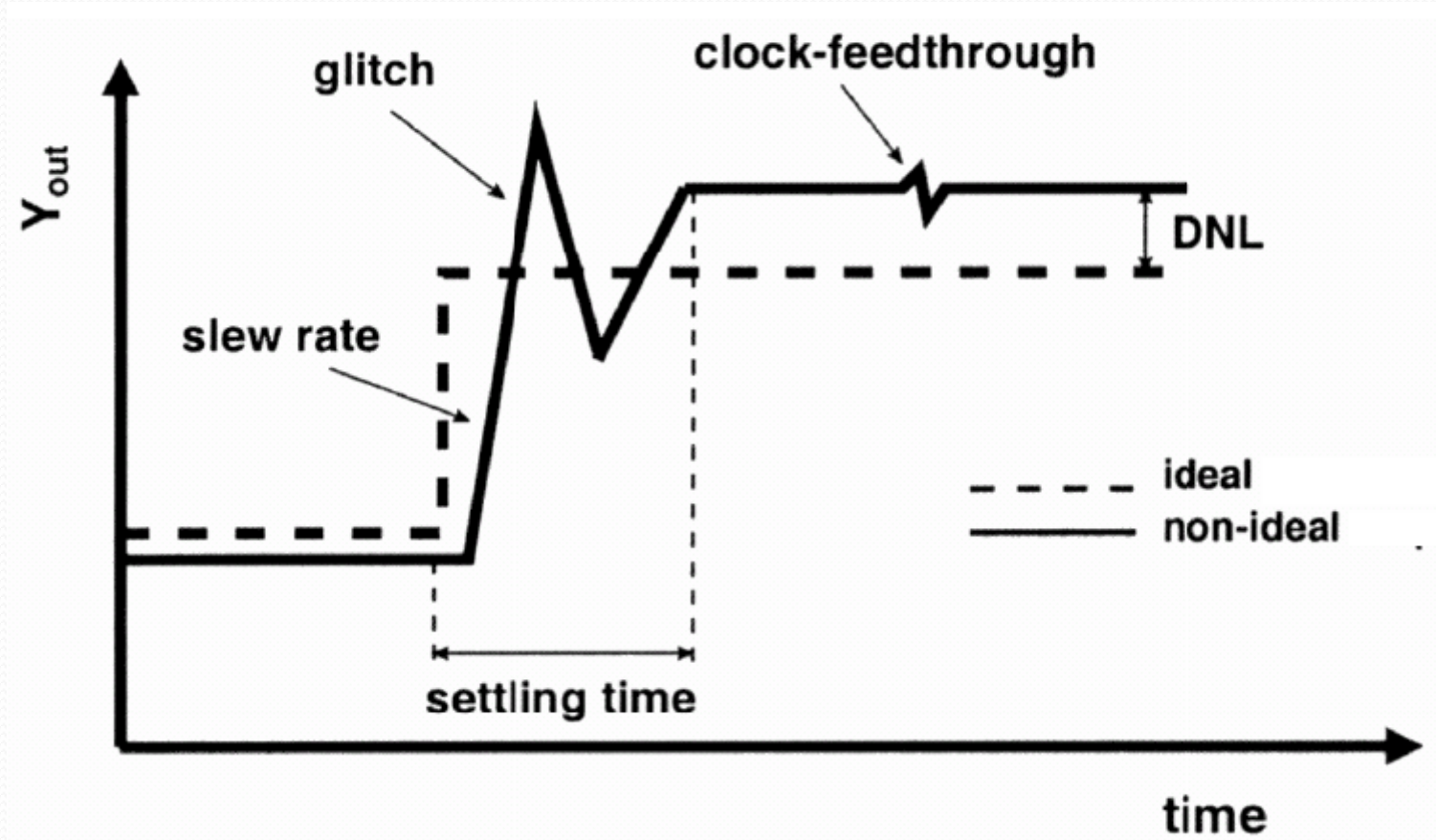
INL **Integral Non Linearity**
DNL **Differential Non Linearity**
(both are expressed in terms of LSB's)



⇒ monotonicity is guaranteed only in a thermometer-coded DAC

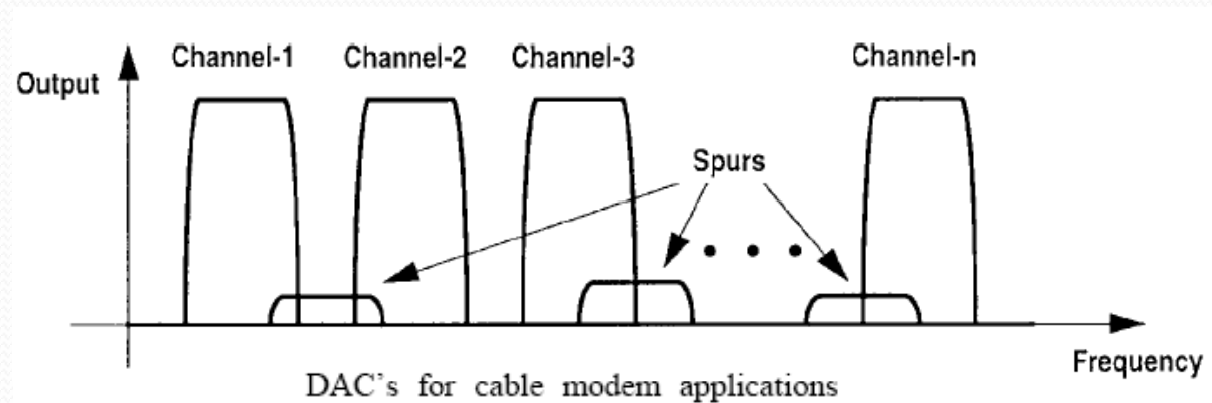
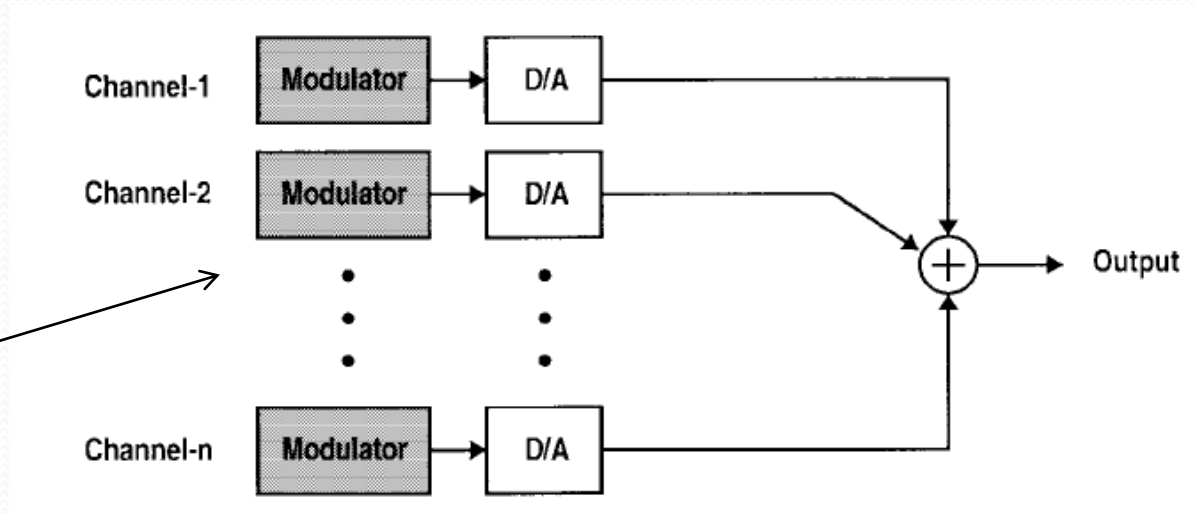
offset error

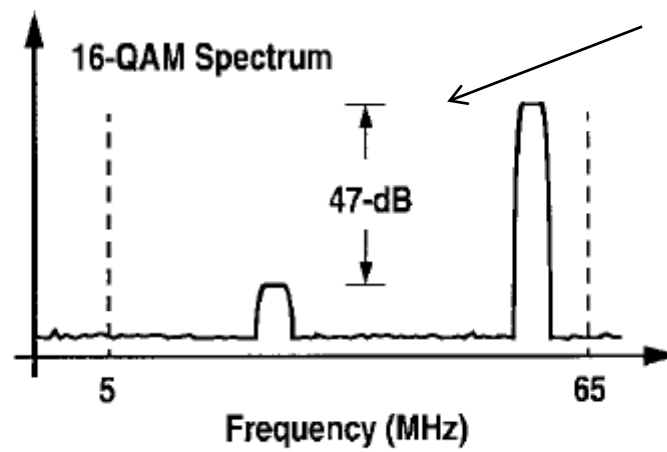




DACs for frequency-domain applications

QAM, QPSK

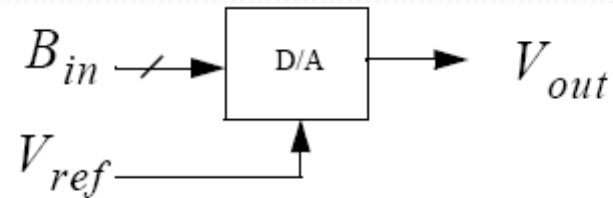




**10-bit resolution
(difficult to achieve at HF)**

16-QAM spectrum including an aliased harmonics 47 dB below fundamental

D/A Converter Basics.



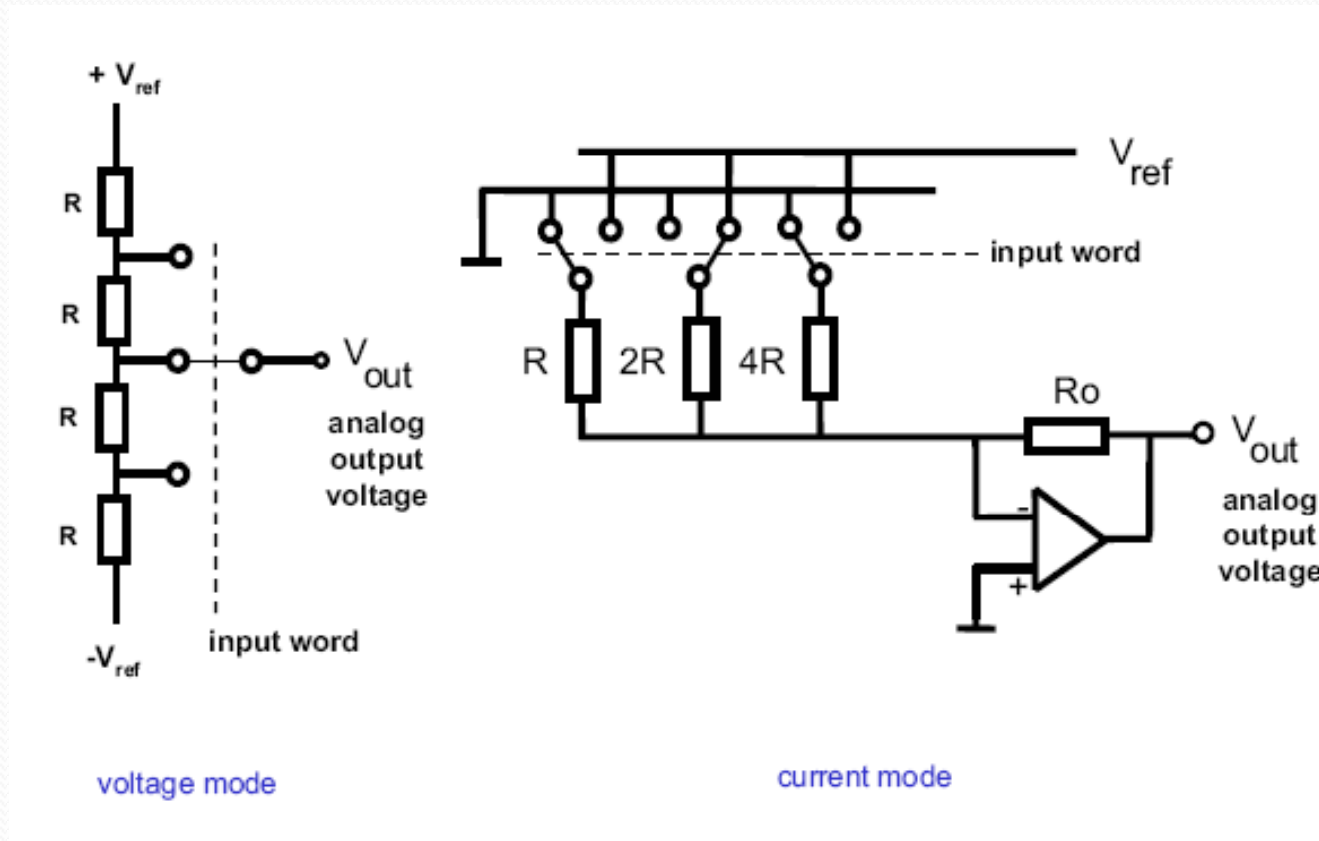
- B_{in} is a digital signal (or word),

$$B_{in} = b_1 2^{-1} + b_2 2^{-2} + \dots + b_N 2^{-N}$$

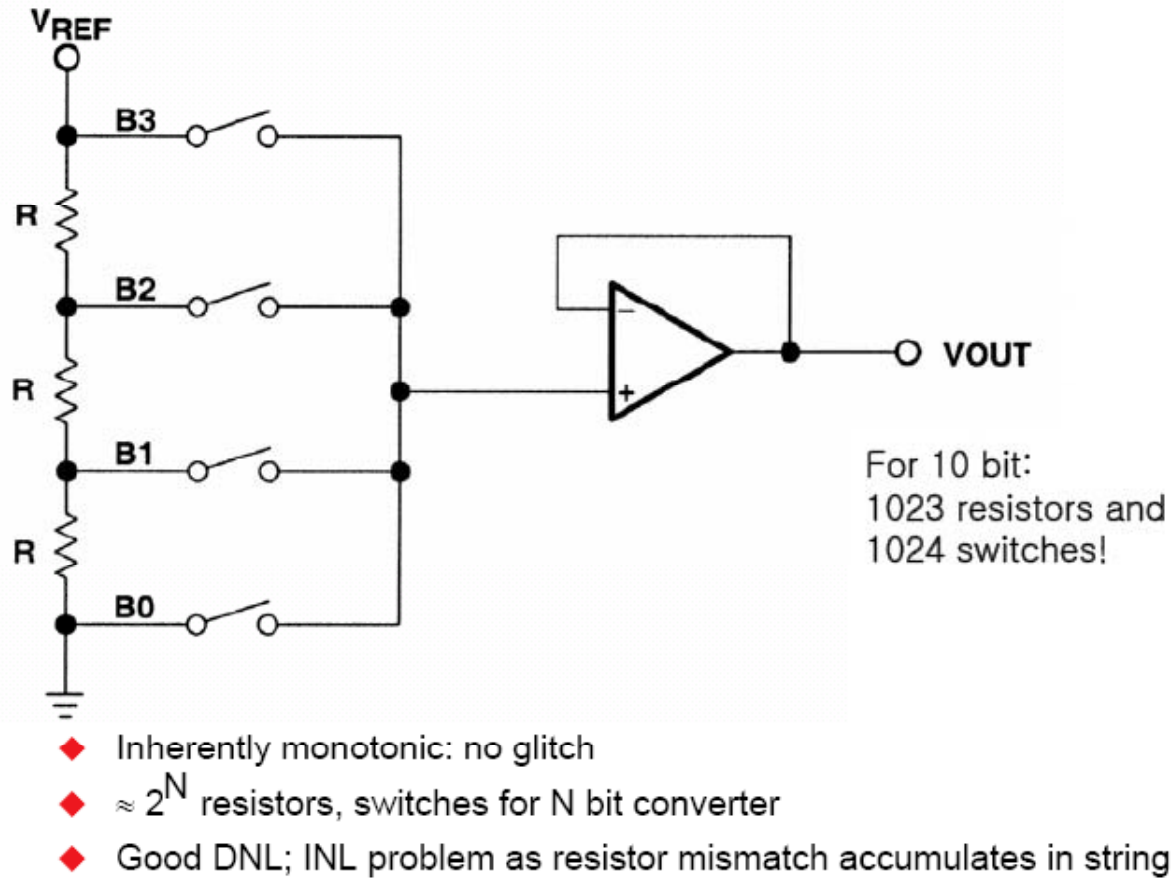
- b_i equals a “1” or a “0” (i.e. a binary digit).
- V_{ref} — an analog reference; V_{out} — output .

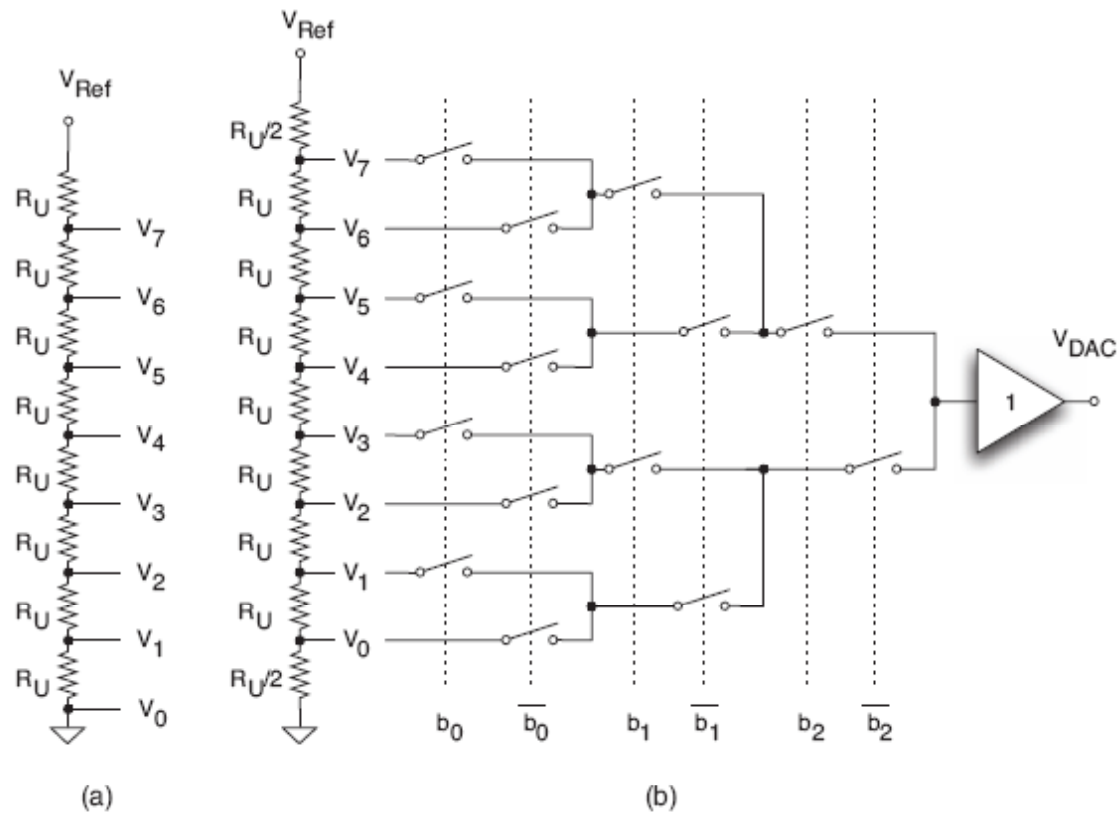
$$V_{out} = V_{ref}(b_1 2^{-1} + b_2 2^{-2} + \dots + b_N 2^{-N})$$

D/A Converter (Nyquist Rate)

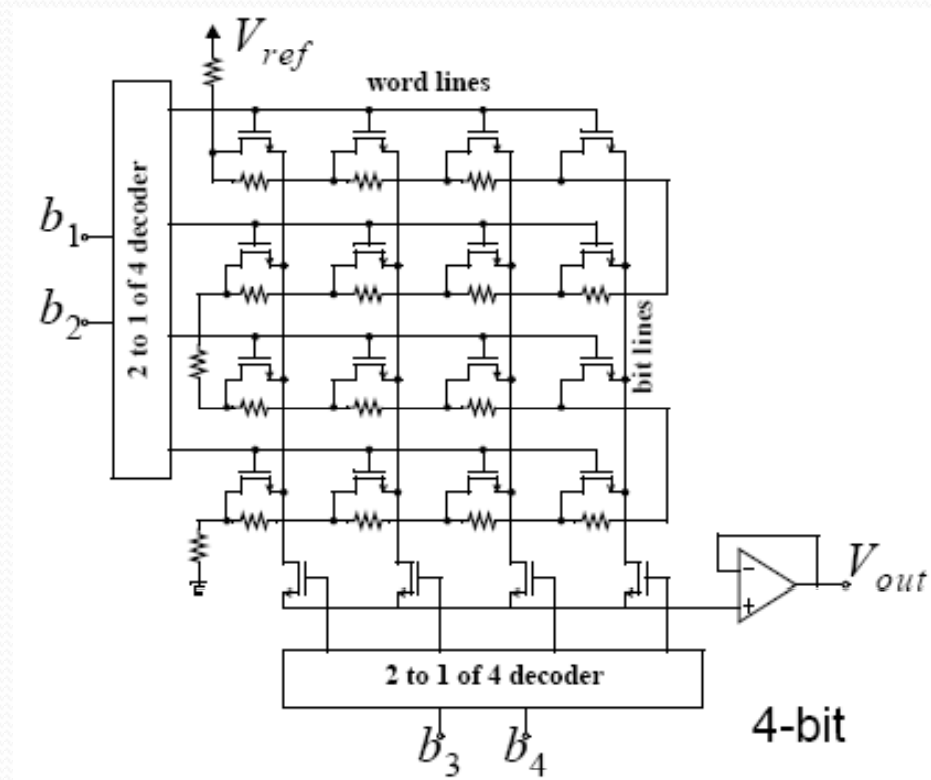
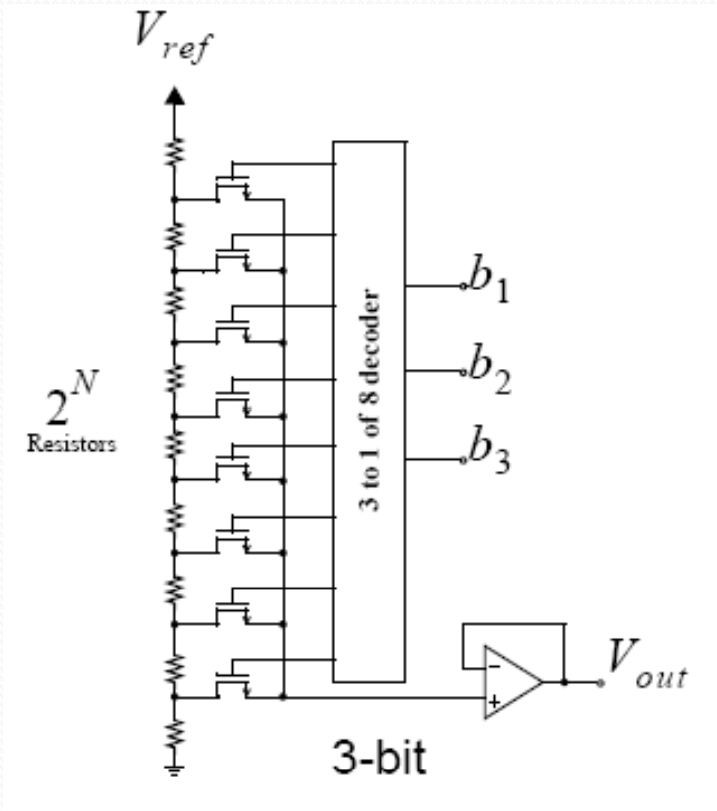


Thermometer Resistive-Divider D/A





(a) Resistive divider. (b) Resistive divider with 1/2 LSB offset, Switch tree selector and output buffer.



Binary			Thermometer						
A	B	C	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇
0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	1	1	0	0	0	0	0
0	1	1	1	1	1	0	0	0	0
1	0	0	1	1	1	1	0	0	0
1	0	1	1	1	1	1	1	0	0
1	1	0	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1

$$T_1 = A + B + C$$

$$T_2 = A + B$$

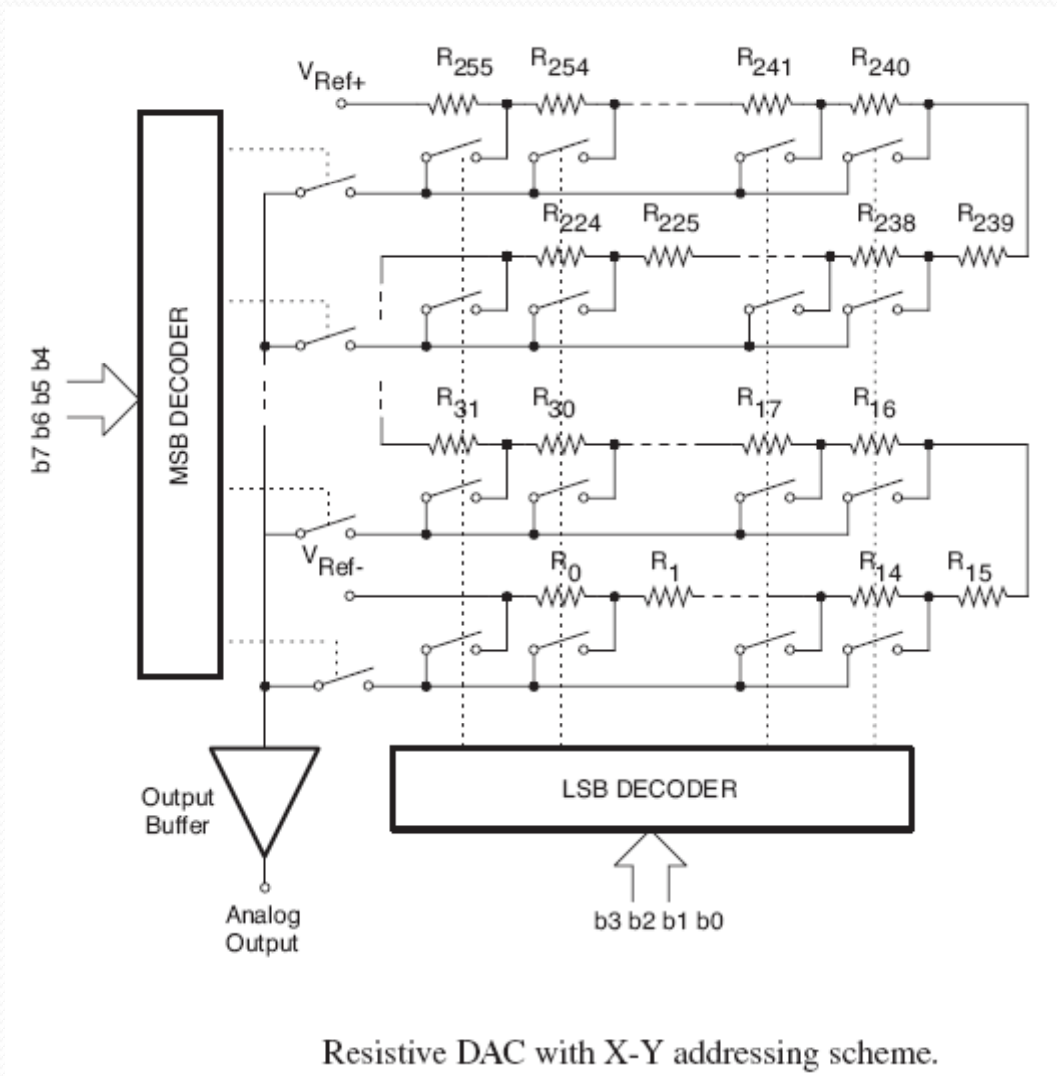
$$T_3 = A + BC$$

$$T_4 = A$$

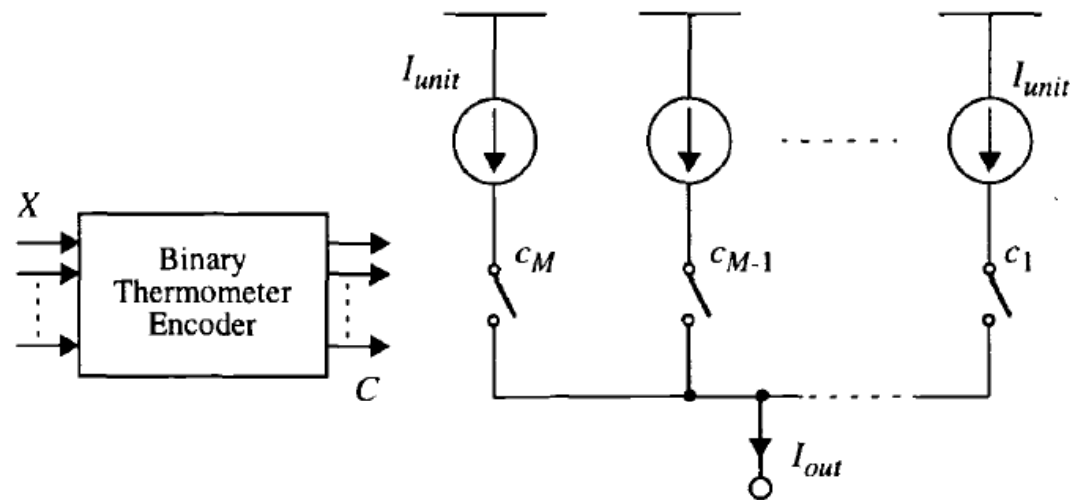
$$T_5 = A(B + C)$$

$$T_6 = AB$$

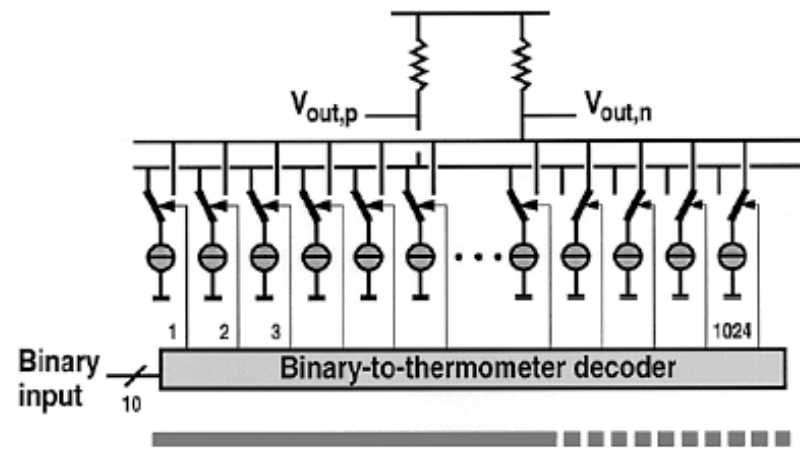
$$T_7 = ABC$$



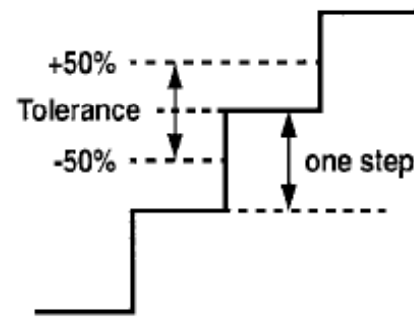
Thermometer Code Current Steering DAC



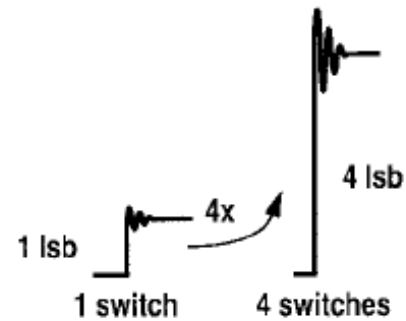
- reference elements are all equally large and their matching becomes simpler than for the binary case
- even if the matching is within a 50% margin, the converter is still monotonic (spread < DNL of 1 LSB)
- for N bits, $2^N - 1$ current sources are needed



(a)



(b)



(c)

Matching and glitch advantages of a thermometer-coded DAC

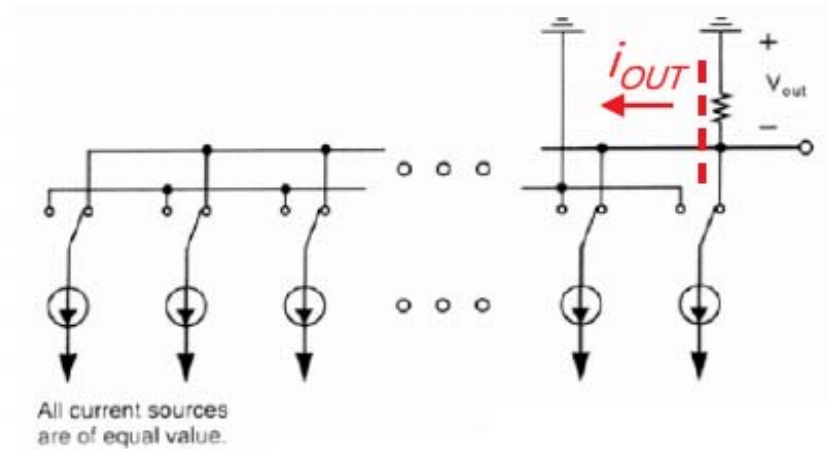
pros:

- monotonic by construction: analog output always increases with digital input
- no problem at mid-code transition (ex: 0111 1111 \rightarrow 1000 0000) as only ONE current source is switched. Glitches are reduced.

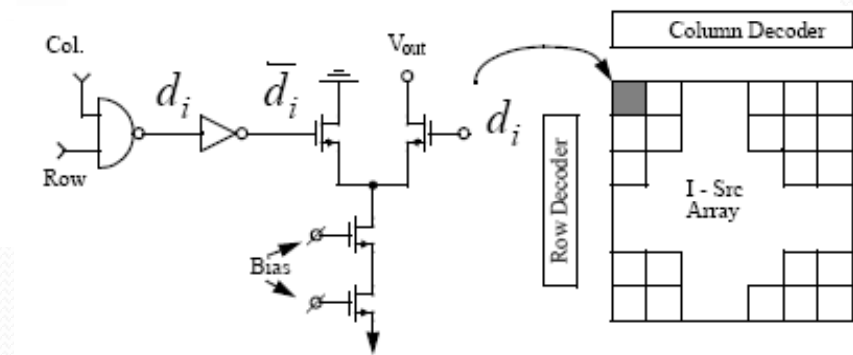
cons:

- design complexity: $2^N - 1$ current sources. Large output capacitance (Ex: 12 bits, 4095 current sources and switches)
- large area
 - matching problems
 - larger timing uncertainty of the switches.

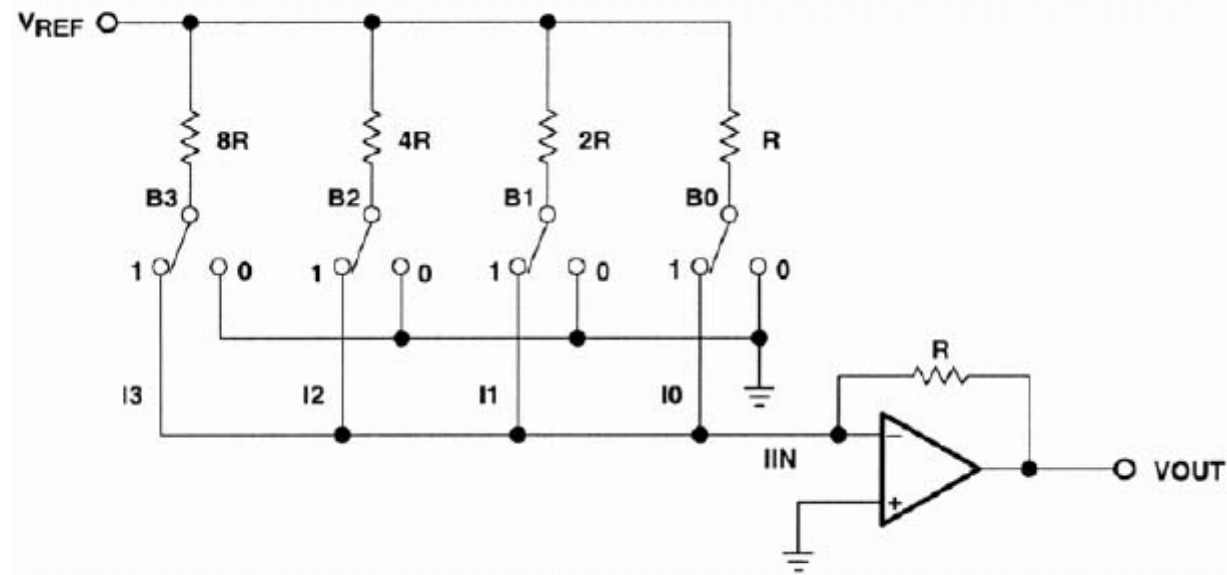
- ◆ All current sources of equal value
- ◆ Reduces glitches



- ◆ Switch with differential pair Q1/Q2, cascode Q3:
 - V_{DS} of current source Q4 remains constant



Binary-Weighted Resistor DAC

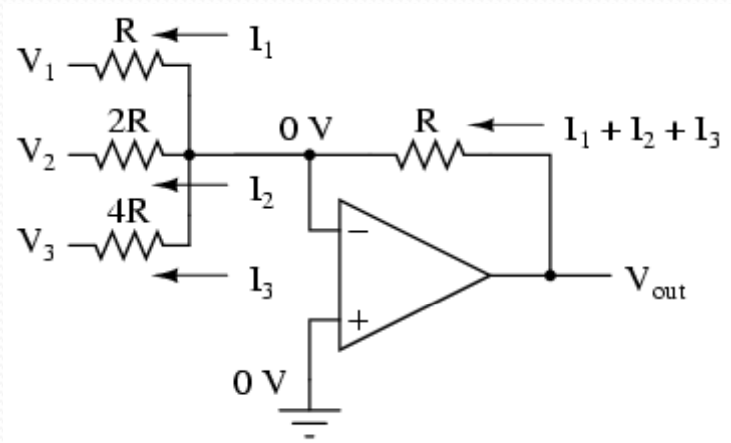


Matching

- ◆ Resistor values vary over wide range: R to $2^{(N-1)}R$ for N -bit converter.
- ◆ Difficult to acquire accurate matching over wide range.

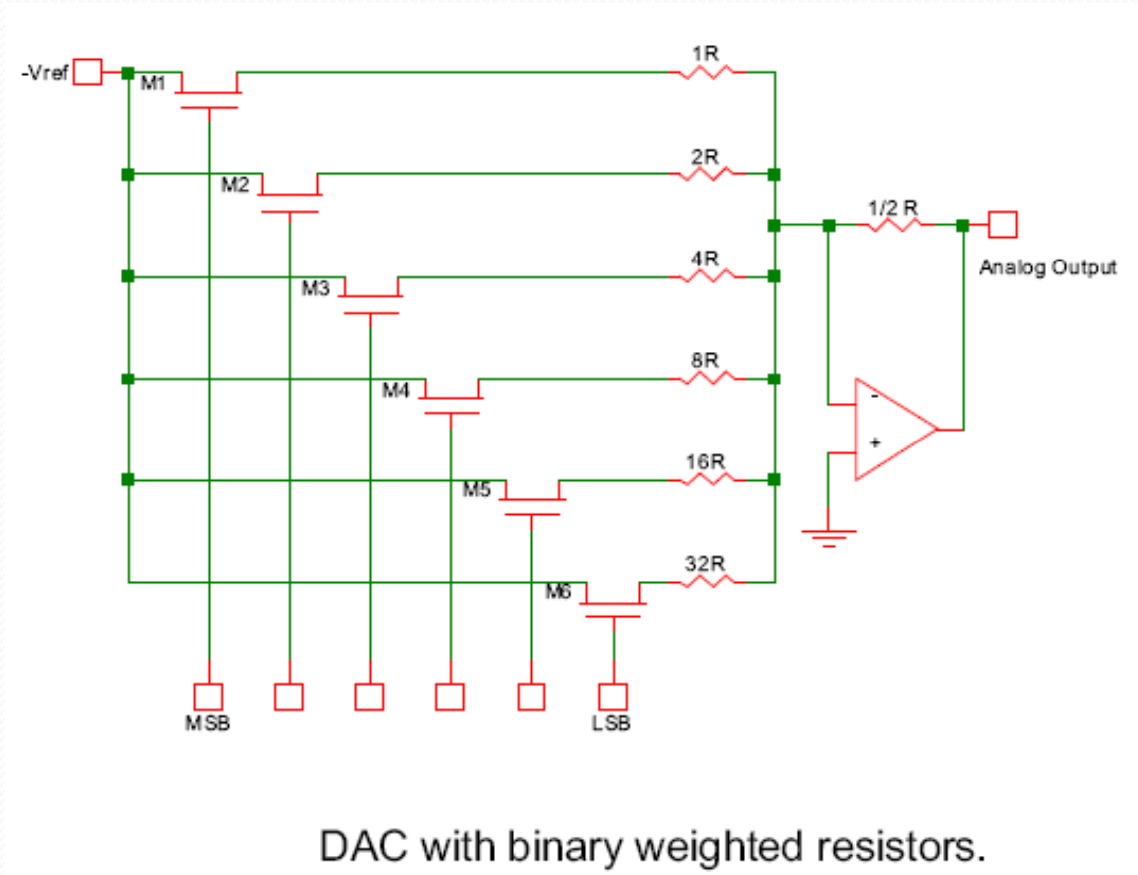
• no guarantee of monotonicity!

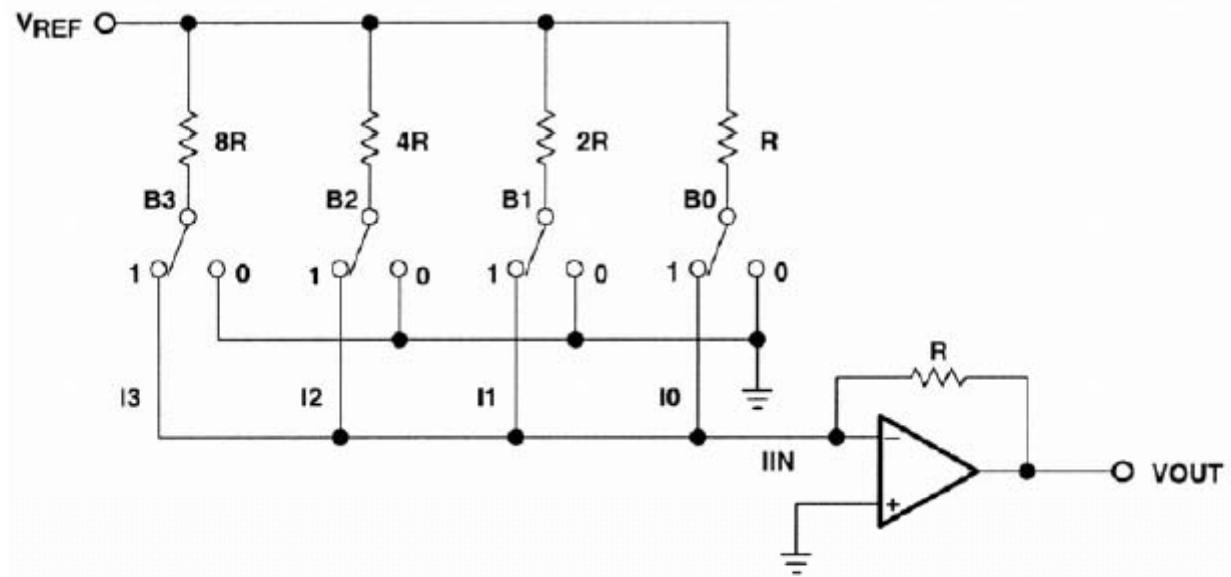
$$V_{out} = -\frac{V_{ref}}{2^{n-1}} (2^{n-1} b_{n-1} + 2^{n-2} b_{n-2} + \dots + 2b_1 + b_0)$$



$$V_{out} = -\left(V_1 + \frac{V_2}{2} + \frac{V_3}{4}\right)$$

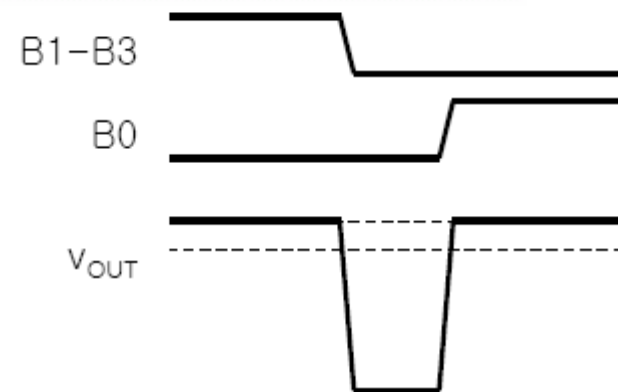
$$V_{out} = -R \left(\frac{V_1}{R} + \frac{V_2}{2R} + \frac{V_3}{4R} \right)$$



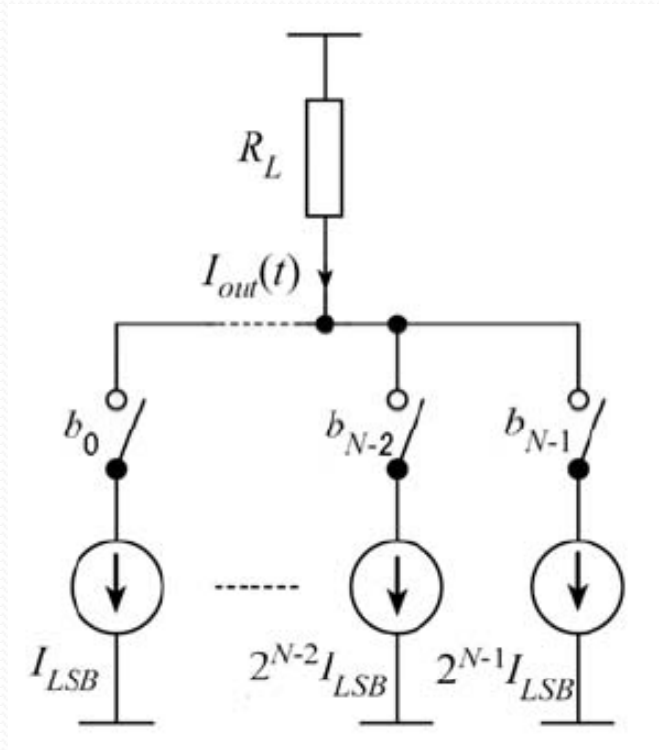


Glitch

- ◆ Consider 0111 to 1000 transition
- ◆ Timing mismatch: glitch



Binary-weighted current-steering D/A



$$\begin{aligned} I_{out(w)} &= 2^{(N-1)} I_{LSB} \cdot b_{N-1} + \dots \\ &+ 2 I_{LSB} \cdot b_1 + I_{LSB} \cdot b_0 \\ &= I_{LSB} \cdot w \end{aligned}$$

digital word

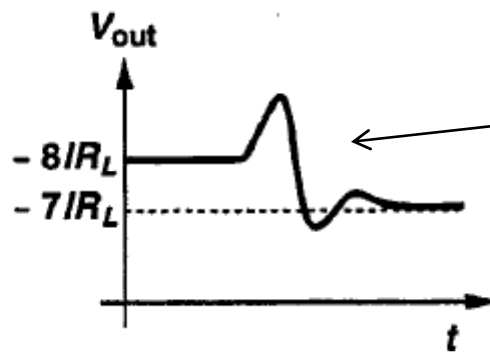
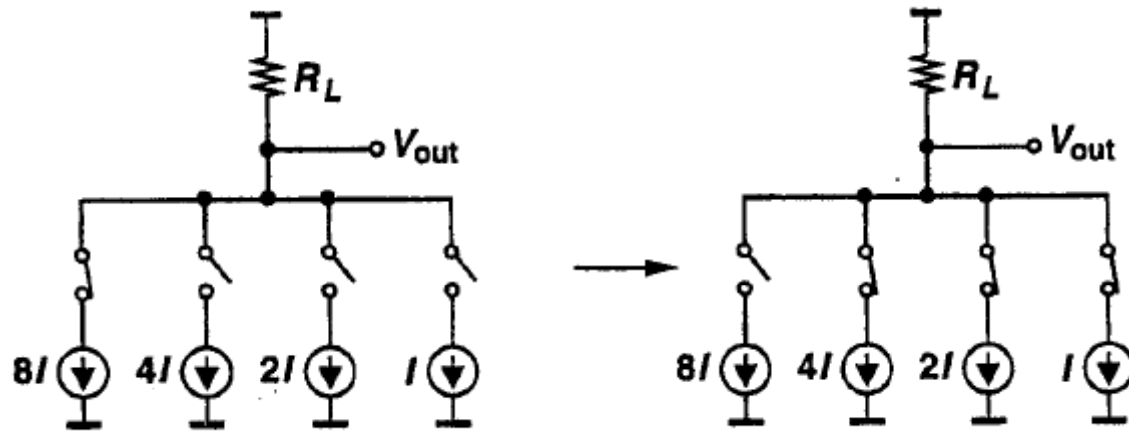
$$w = 2^{(N-1)} \cdot b_{N-1} + \dots + 2 \cdot b_1 + b_0 = \sum_{m=0}^{N-1} 2^m \cdot b_m$$

pros:

- simple circuit: Ex. for 12-bit converter, only 12 current sources and equal number of switches
- no need for decoding logic: all switches controlled directly by input bits.

cons:

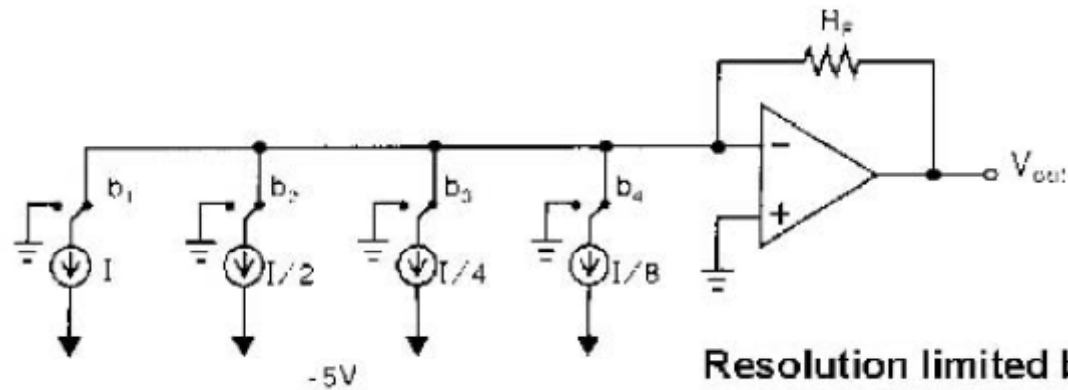
- good matching required: MSB has to be matched within $\frac{1}{2}$ LSB to the sum of all other bits (ex, difficult to guarantee for 12 bits)
- DAC monotonicity is not ensured. Ex mid-code transition (ex: 0111 1111 \rightarrow 1000 0000) missing-code (DC) + glitch (transient)
- fully binary DACs commonly limited to 10 bits



clock skew during
1000 → 0111
(all switches change)

Glitch impulse in a current-steering DAC.

Binary-weighted current-steering D/A



Resolution limited by
Mismatch in the
Current sources !

Glitches !

- no guarantee of monotonicity!

R-2R Divider D/A

- to decrease resistor spread

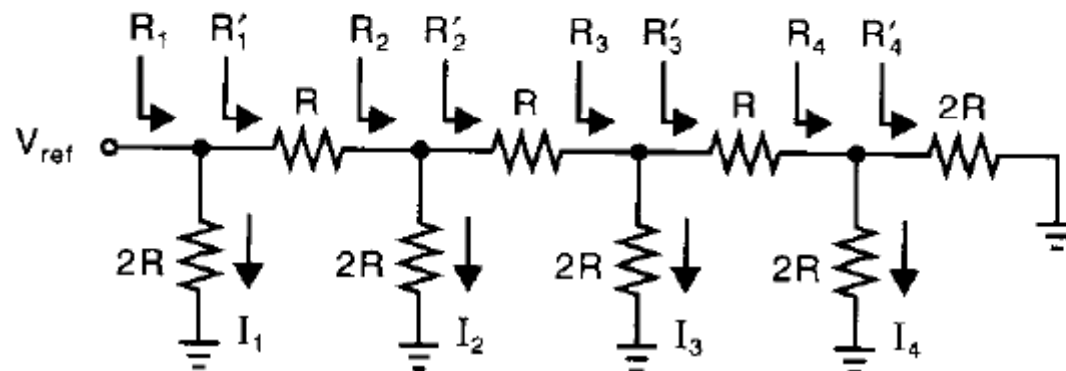
$$R'_4 = 2R$$

$$R_4 = 2R \parallel 2R = R$$

$$R'_3 = R + R_4 = 2R$$

$$R_3 = 2R \parallel R'_3 = R$$

and so on. Thus, $R'_i = 2R$ for all i . This result gives the following current relationships:



R-2R resistance ladder.

$$I_1 = \frac{V_{ref}}{2R}$$

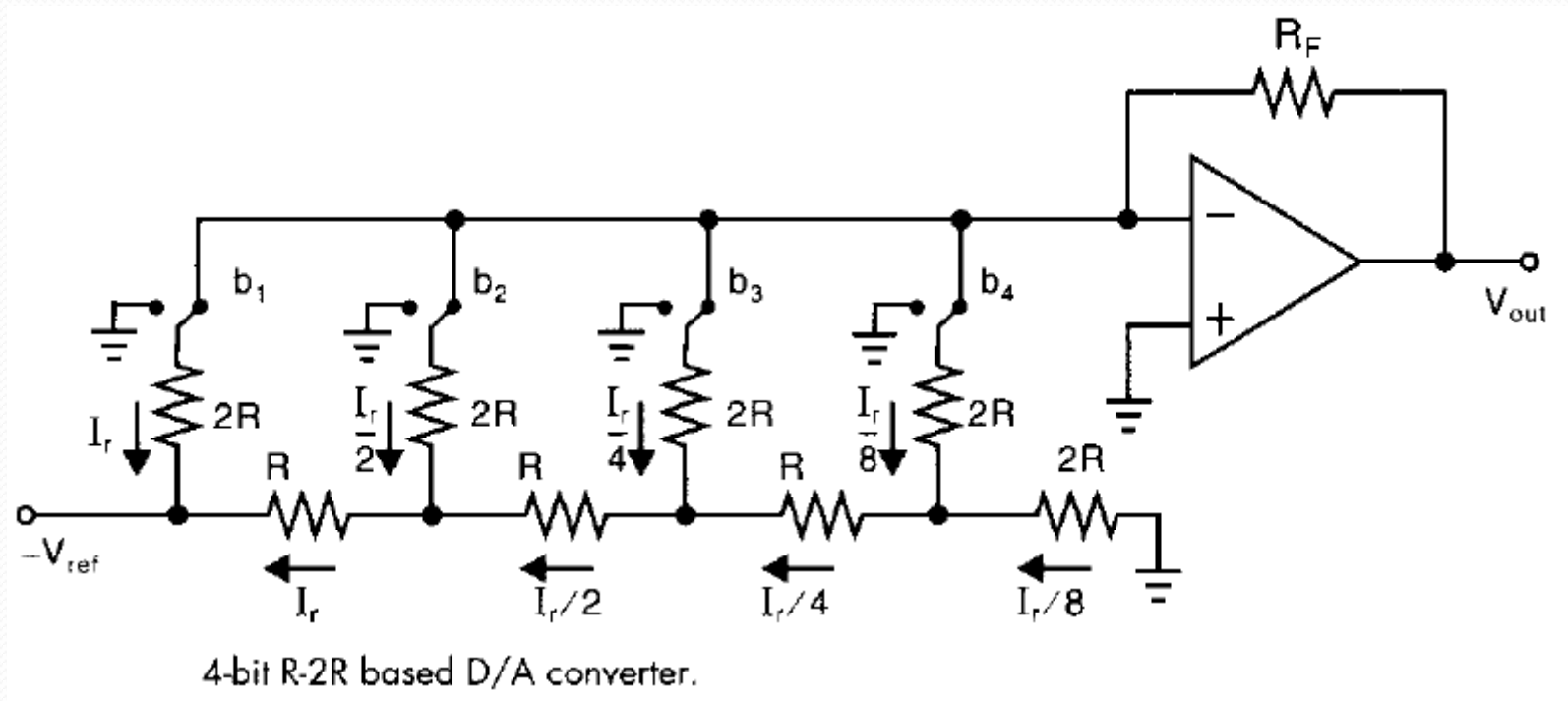
$$I_2 = \frac{V_{ref}}{4R}$$

$$I_3 = \frac{V_{ref}}{8R}$$

$$I_r = V_{ref}/(2^i R)$$

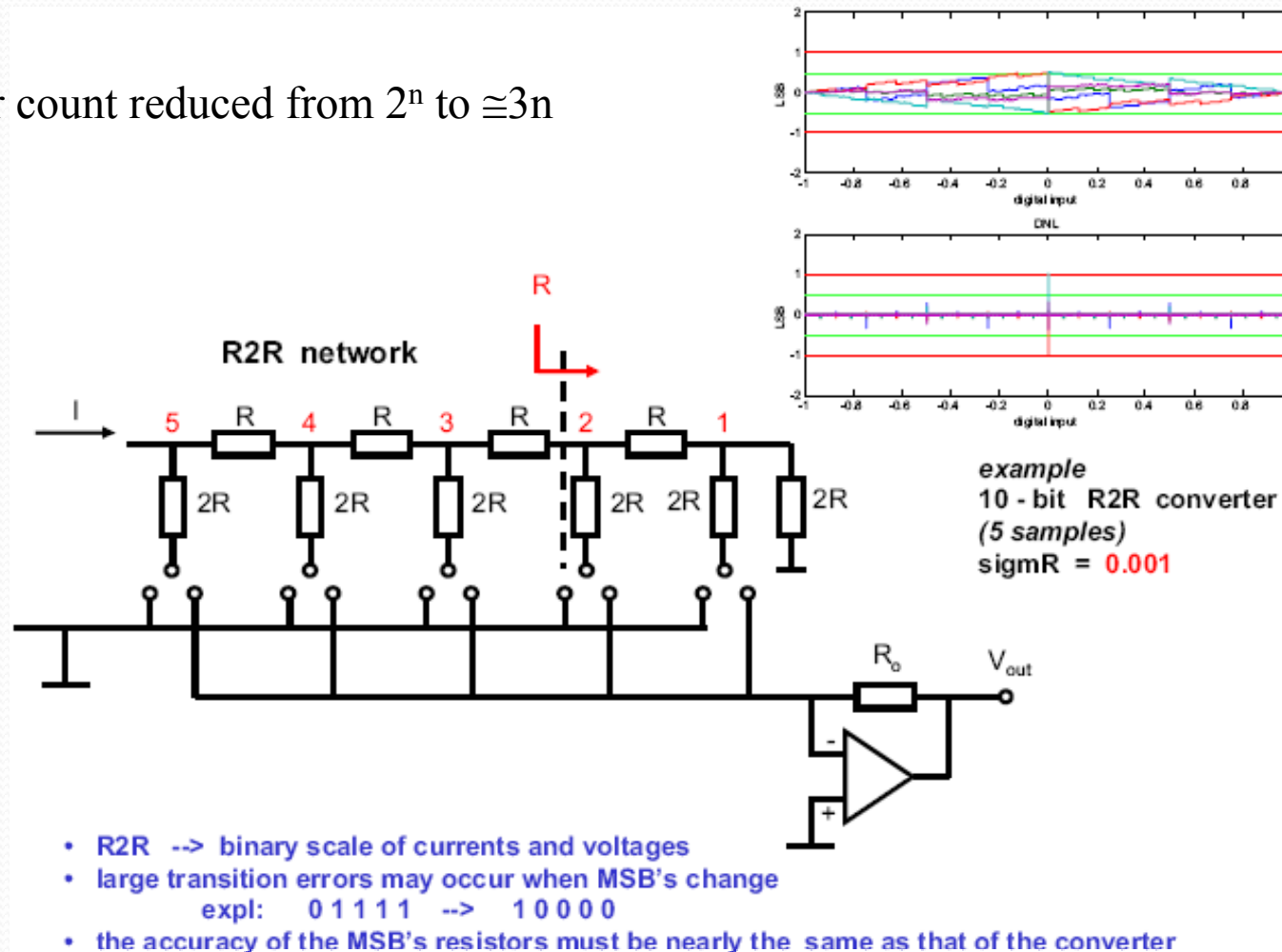
$$V_{out} = R_F \sum_{i=1}^N \frac{b_i I_r}{2^{i-1}} = V_{ref} \left(\frac{R_F}{R} \right) \sum_{i=1}^N \frac{b_i}{2^i}$$

R-2R Divider D/A

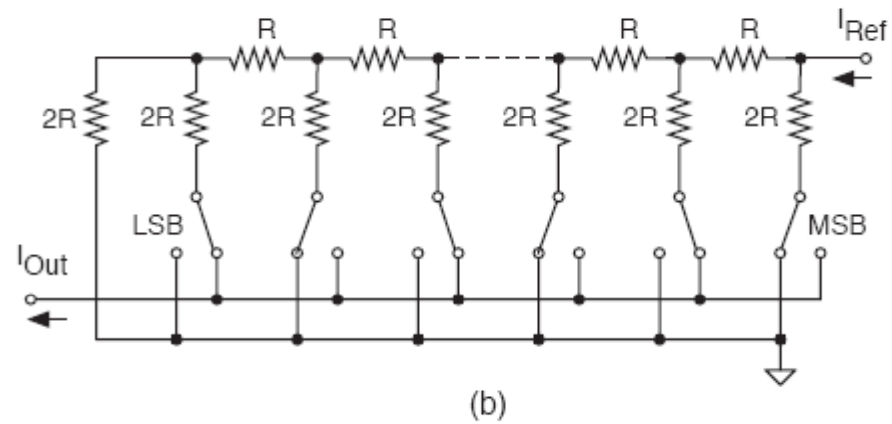
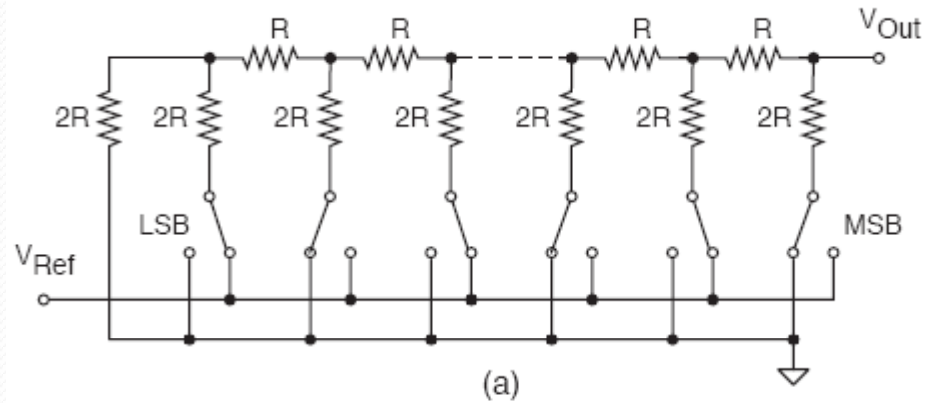


R-2R Divider D/A

- resistor count reduced from 2^n to $\cong 3n$



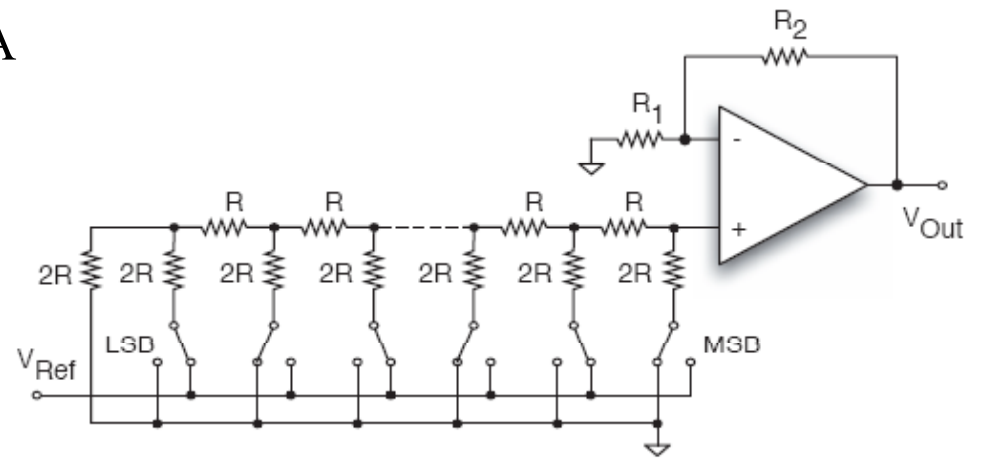
R-2R Divider D/A



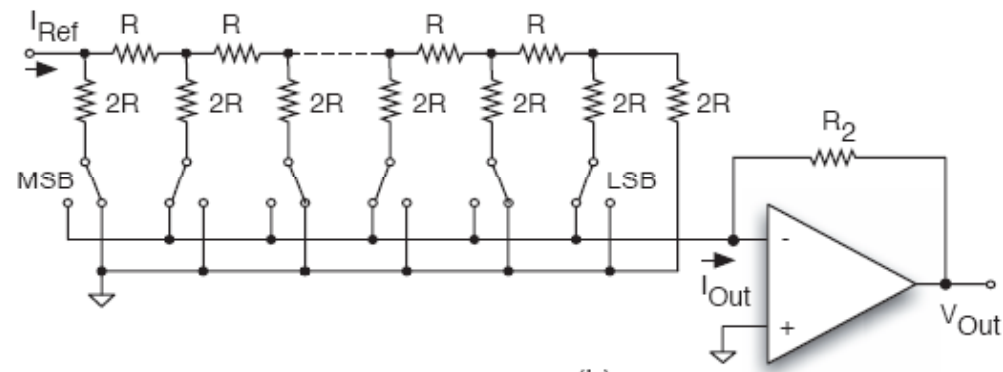
(a) Voltage mode R-2R ladder network. (b) R-2R current mode ladder networks.

$$V_{Out} = \frac{V_{Ref}}{2} b_{n-1} + \frac{V_{Ref}}{4} b_{n-2} + \dots + \frac{V_{Ref}}{2^{n-1}} b_1 + \frac{V_{Ref}}{2^n} b_0$$

R-2R Divider D/A

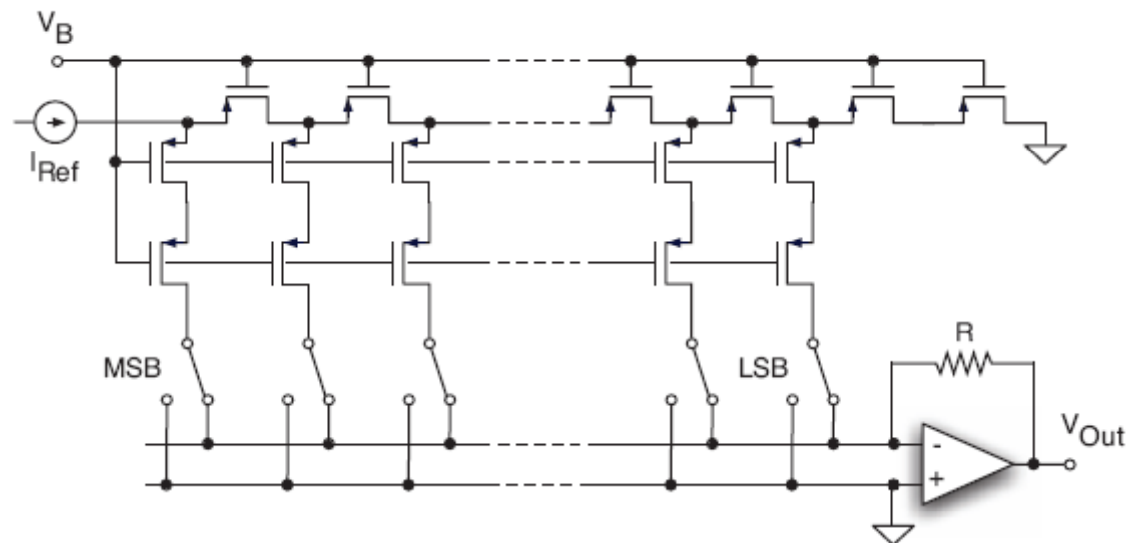


(a)



(b)

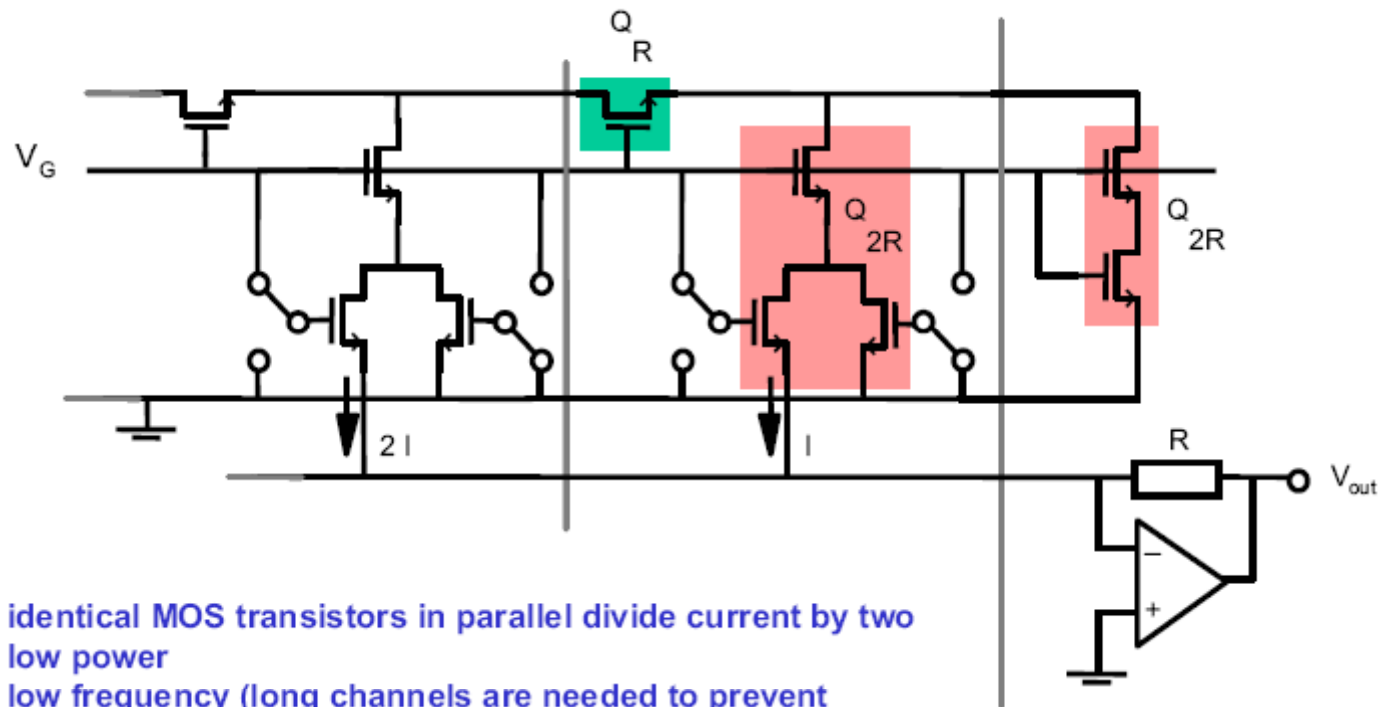
(a) Use of the voltage-mode R-2R ladder network. (b) Use of the current-mode R-2R ladder network in an output voltage DAC.



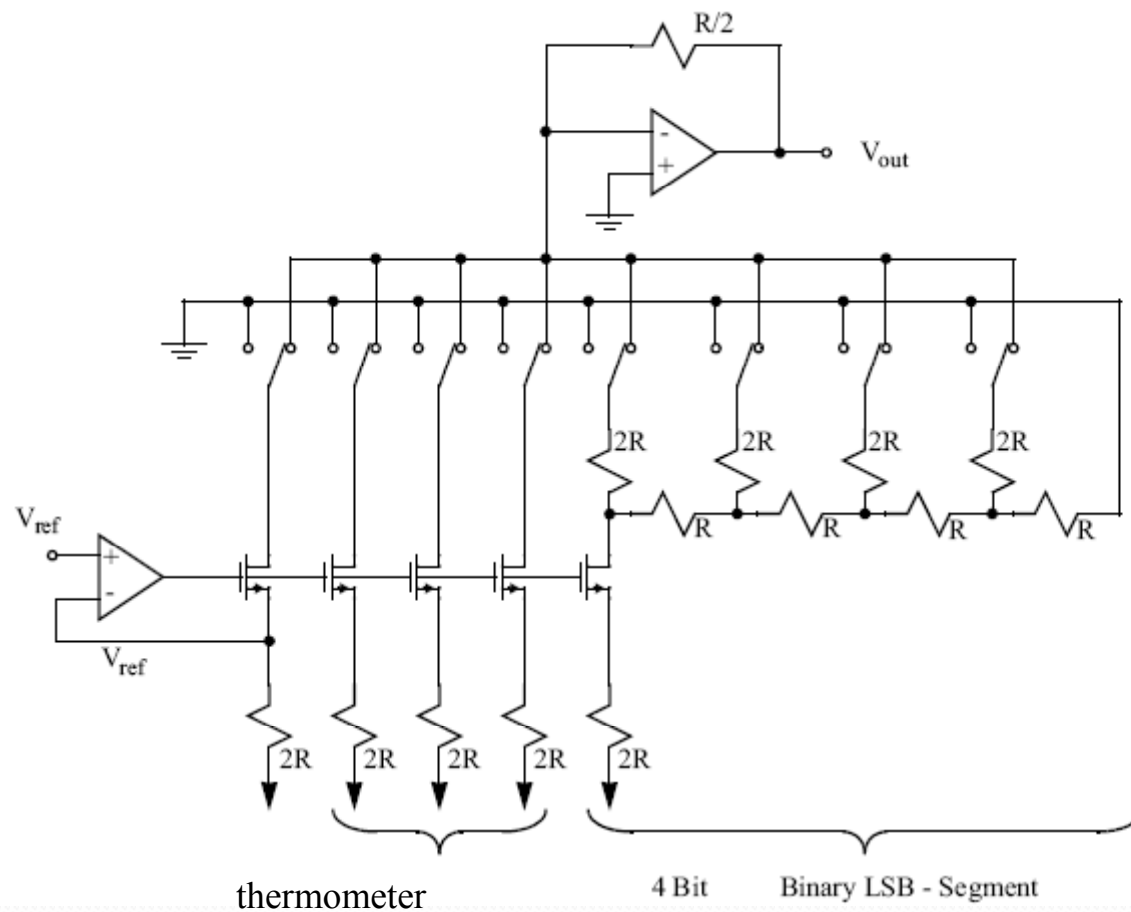
R-2R architecture with MOS transistors replacing the resistors.

- the layout of resistors consumes a relatively large area if the value of unity resistance is large and the used layer has a low specific resistance (like low-resistive poly)
- for medium accuracy it is possible to save silicon area by replacing the resistors with MOS transistors
- all transistors have identical (W/L)


MOSFET - only R2R converter



Segmented D/A



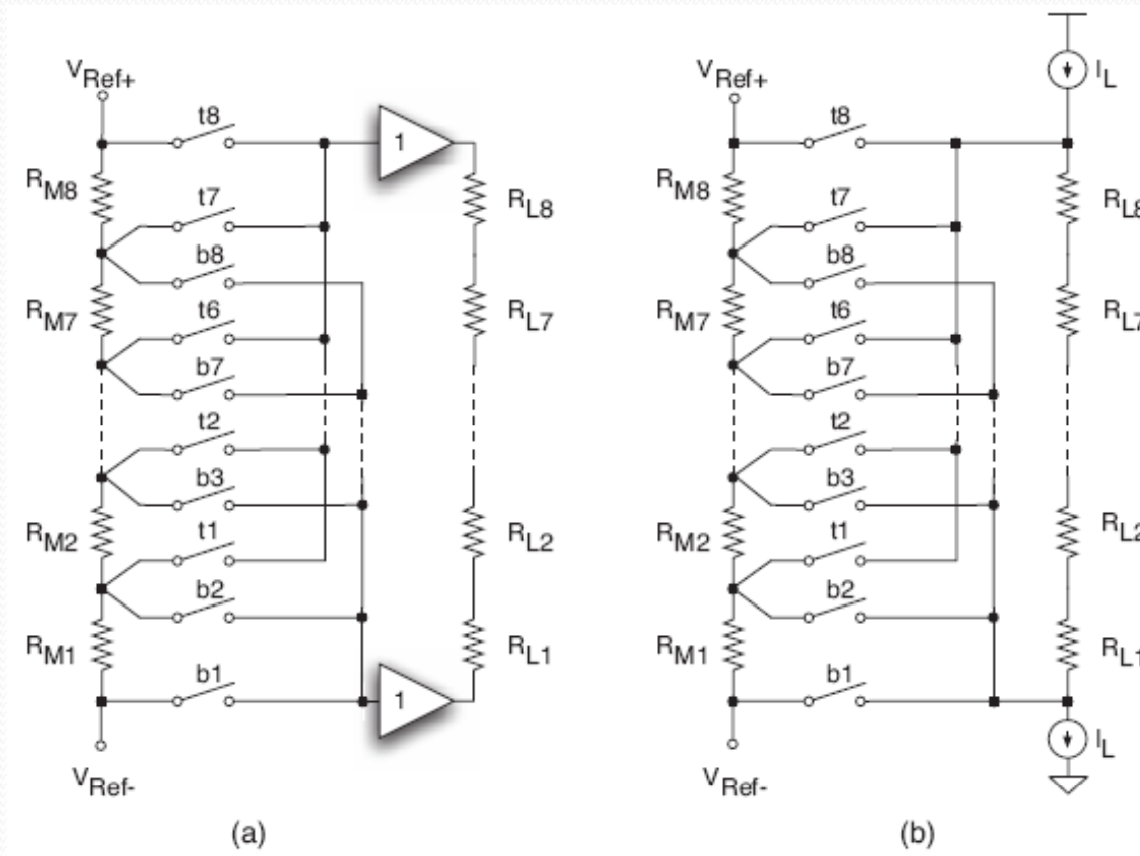
- Combine thermometer and binary
- Accuracy needed for LSB reduced
- Glitches reduced
- Very popular

- 
- **segmentation:** combination of binary and thermometer coding
 - thermometer code is used for MSBs, while binary code for LSBs
 - 0% segmentation = fully binary converter
 - 100% segmentation = a full thermometer code converter


 - low segmentation \Rightarrow circuit simplicity (only a few current sources and switches), but larger DNL
 - large segmentation \Rightarrow lower DNL, but increased problems due to output conductance, capacitance and charge feedthrough, as well as larger area and power consumption.

\Rightarrow optimum segmentation has to be found by the designer!

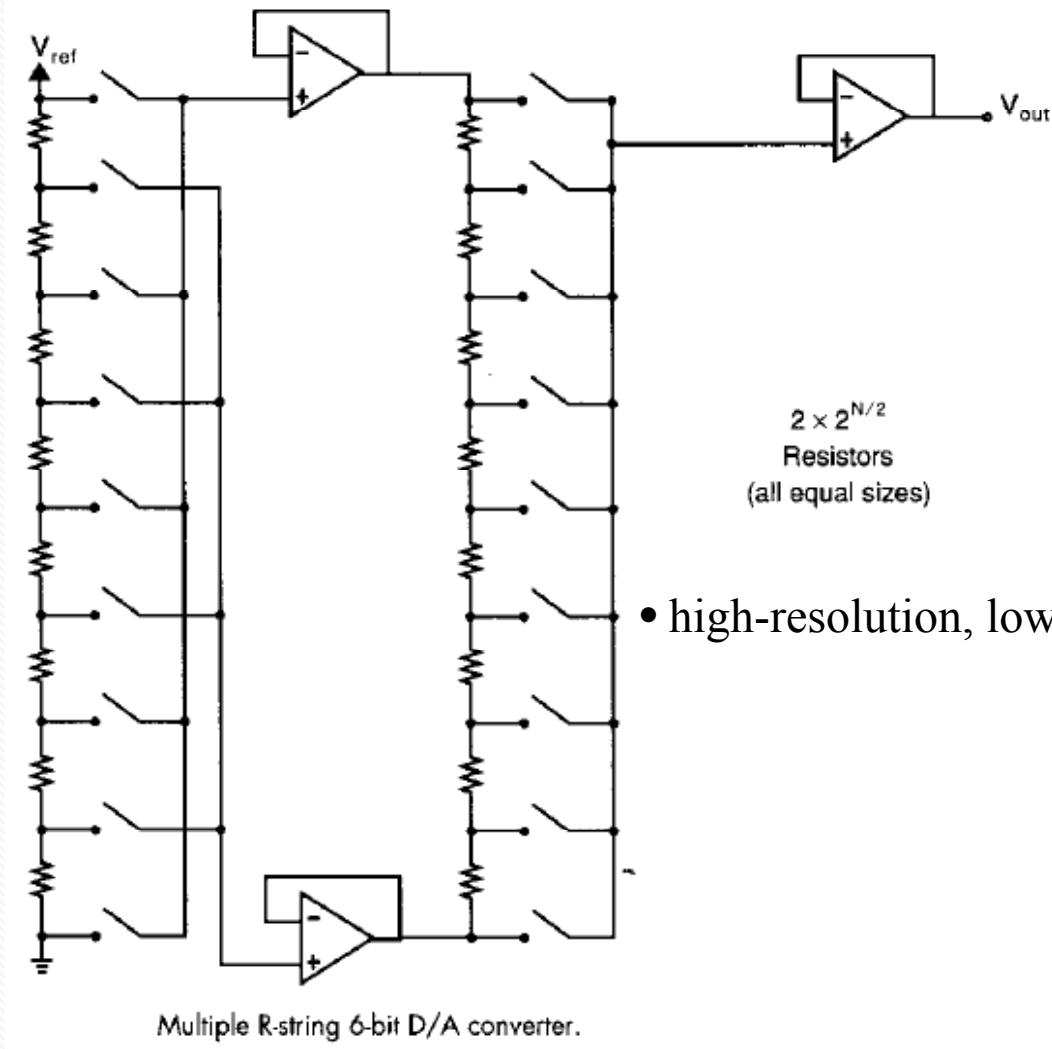
Thermometer-code segmented D/A

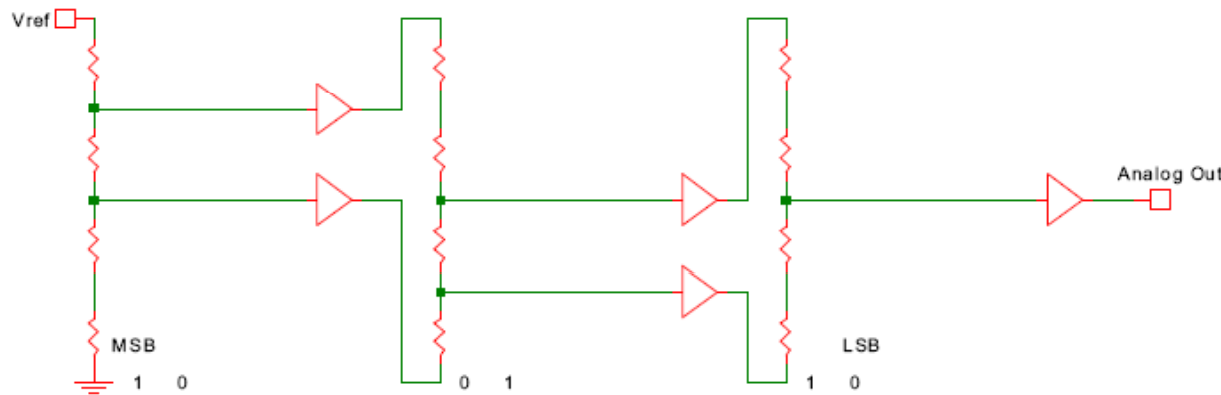
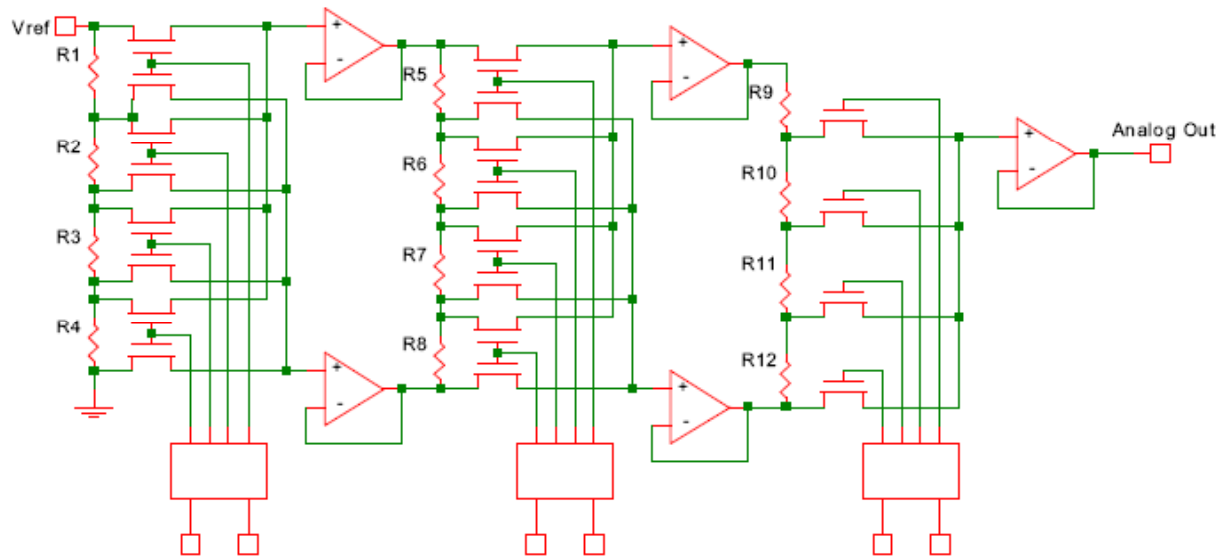


(a) Cascade of two DACs with decoupling buffers. (b) Volt-metric interconnection of the LSB DAC

- 
- Switches select one of the coarse intervals and use it in the fine divider RL1 – RL8
 - Two unity-gain buffers decouple the coarse and the fine dividers. The offset of the buffers must match well ($< \text{LSB}$), and must have an input common mode range V_{REF}
 - Moreover, the input impedance must be very high for sensing the coarse voltages in a volt-metric manner: the output resistance of the buffer must be much less than the total resistance of the *LSB divider*.

Thermometer-code segmented D/A





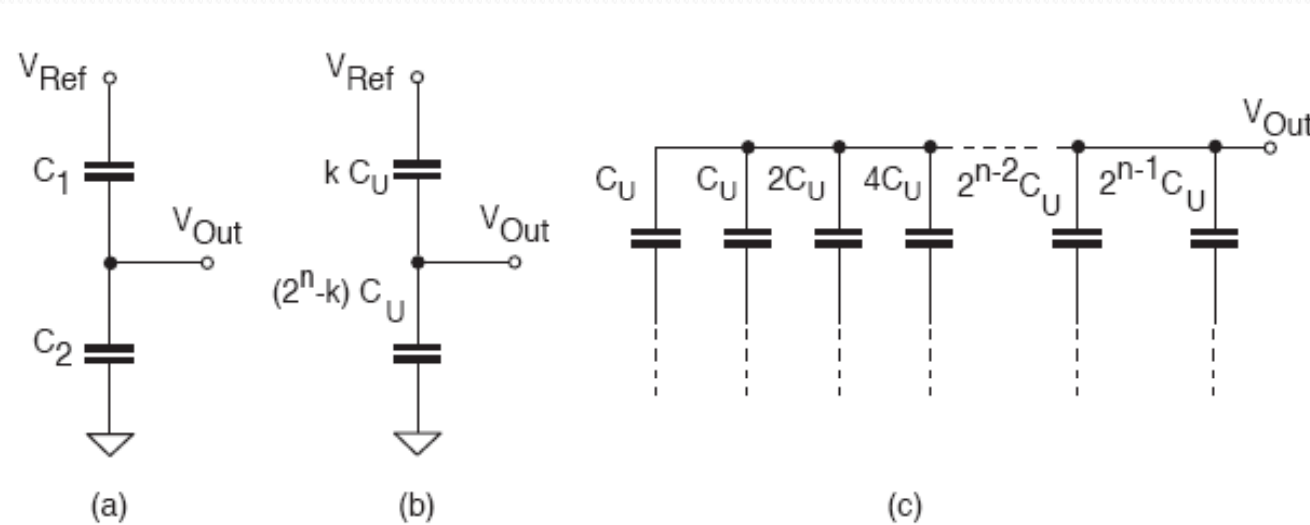
6-bit segmented DAC.

Capacitor-divider D/A

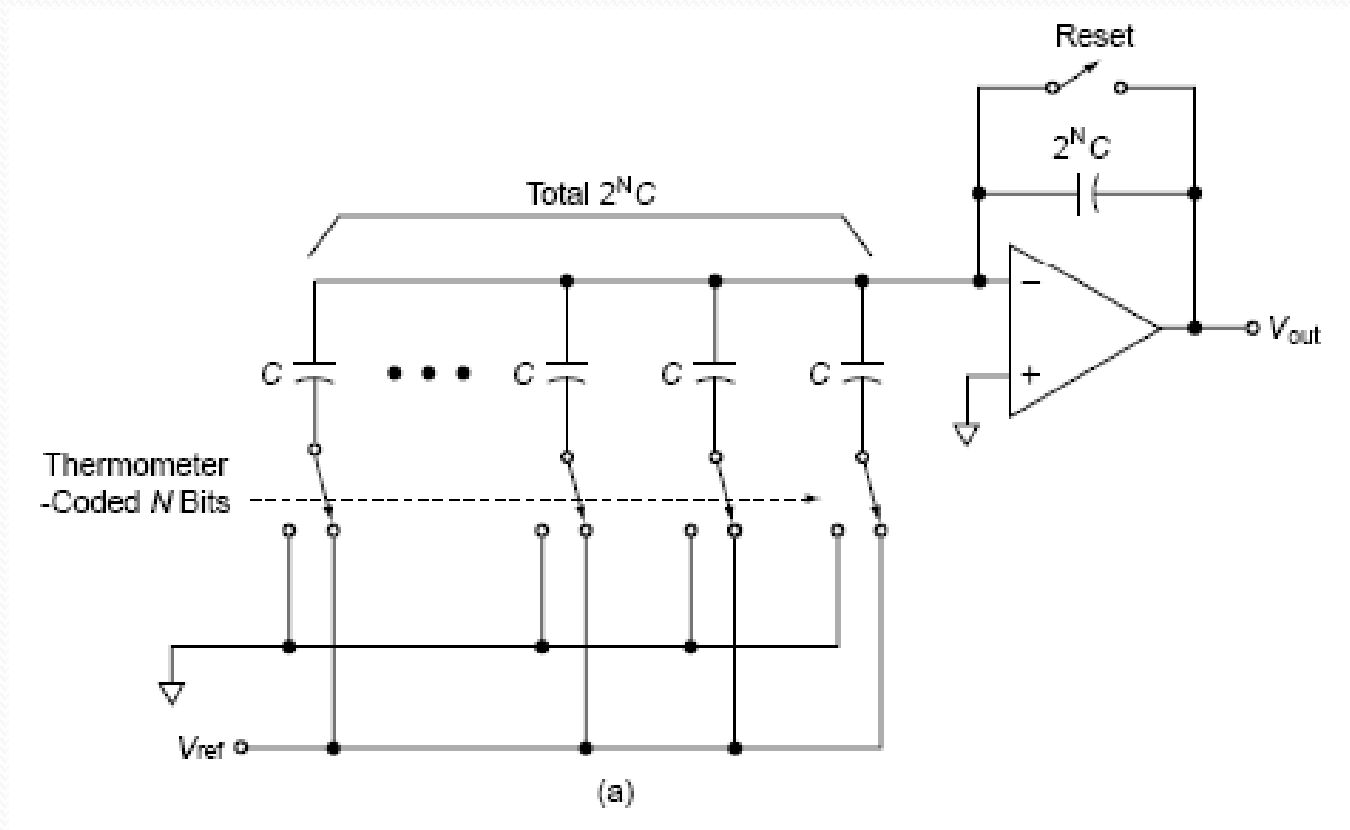
<i>Capacitor</i>	<i>Absolute tolerance</i> %	<i>Matching tolerance</i> %	<i>Voltage coeff</i> ppm/V	<i>Temperat. coeff</i> ppm/°C
<i>Poly - poly</i>	± 10 to 30	± 1 area > 500 μm^2 ± 0.1 area > 10 ⁴ μm^2	10 to 20	20 to 50
<i>Poly - diff</i>	± 10	same	20 to 100	50

Capacitor-divider D/A

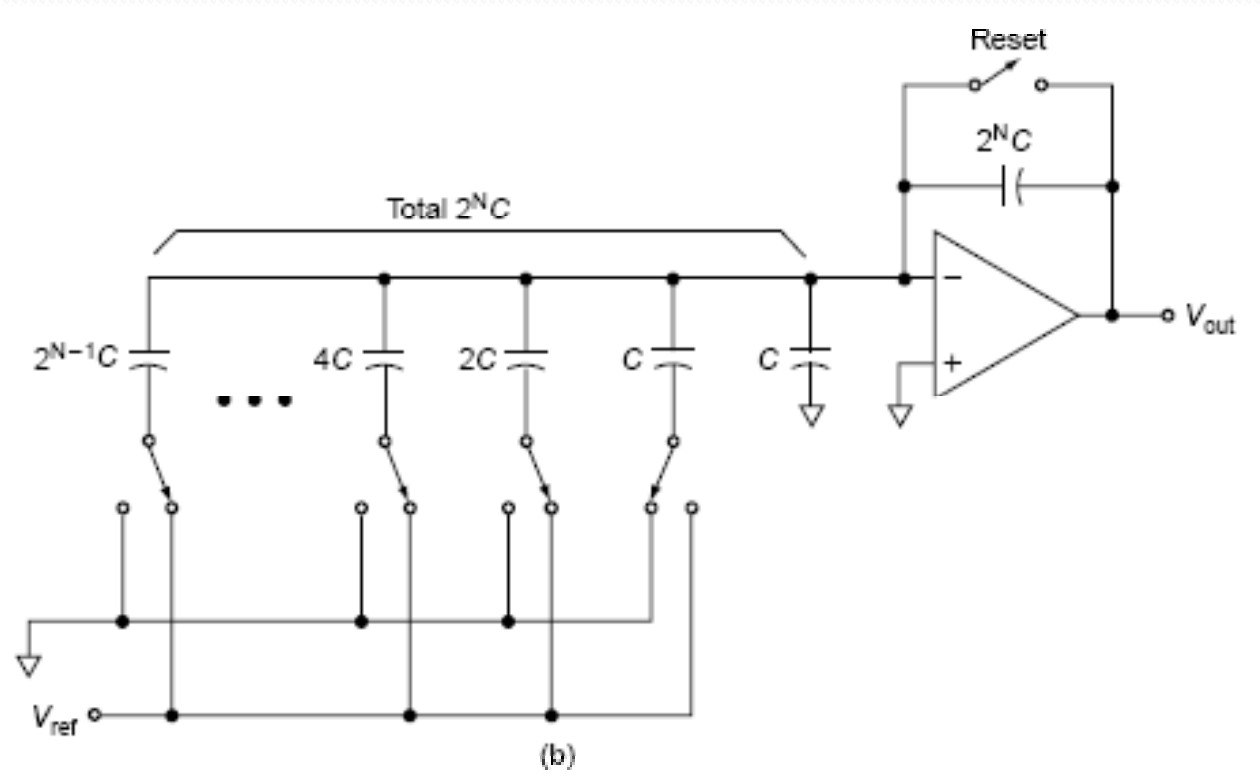
$$V_{Out} = V_{Ref} \frac{C_1}{C_1 + C_2}$$



(a) and (b) Simple capacitor divider. (c) Array of binary weighted capacitors.

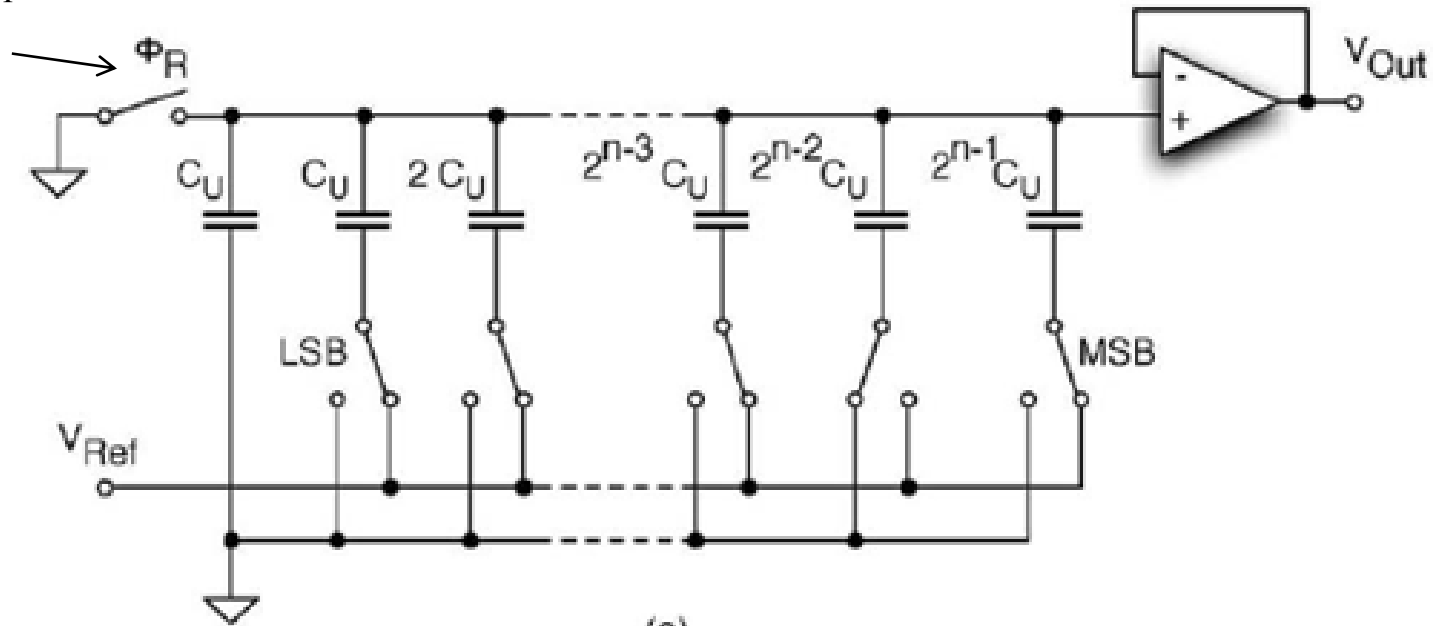


Capacitor-array DACs: (a) thermometer-coded



Capacitor-array DACs (b) binary-weighted.

Reset phase



(a)

(a) n-bit Capacitive divider DAC.

- A drawback of capacitive divider architecture: the number of capacitors increases exponentially with the number of bits.
- The capacitors' count can be reduced by dividing the capacitive array into two parts separated by an attenuation capacitance, C_A .

The series of attenuation capacitance and the entire right array must equal the C_U

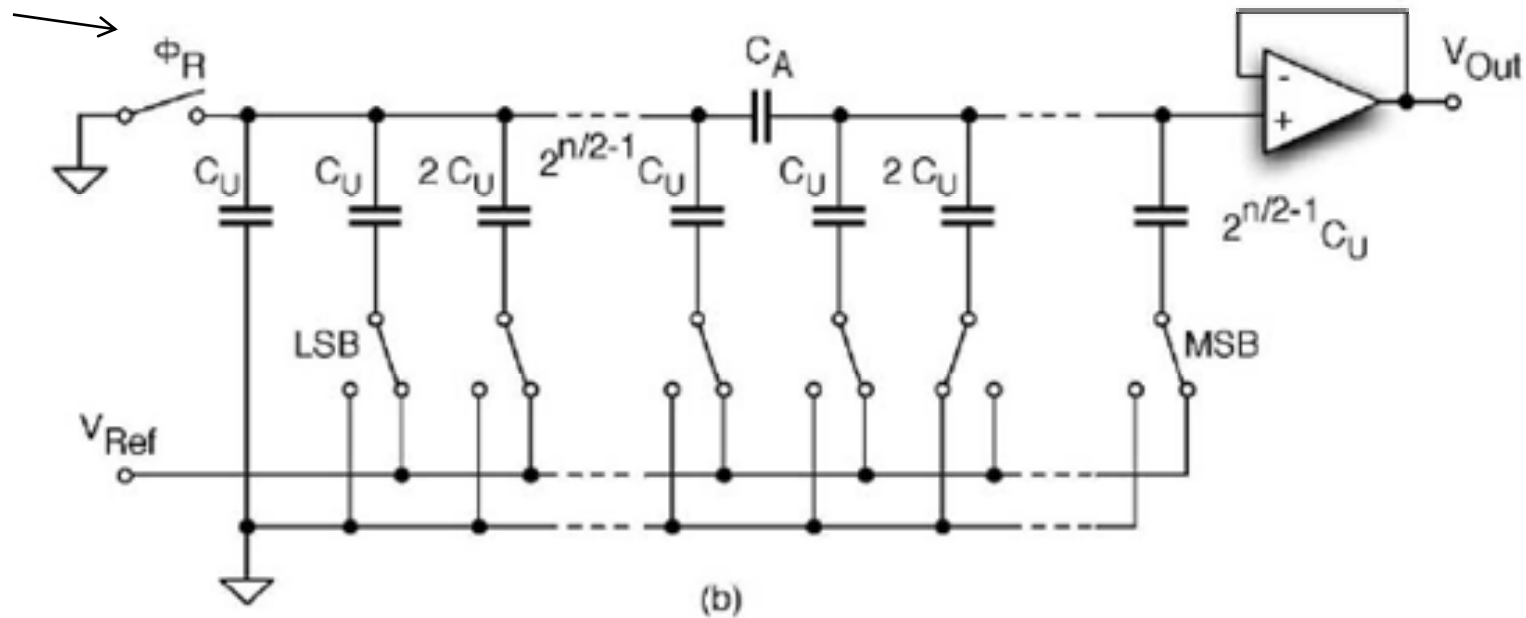
$$\frac{C_A \cdot 2^{n/2} C_U}{C_A + 2^{n/2} C_U} = C_u$$

which yields the value of C_A

$$C_A = \frac{2^{n/2}}{2^{n/2} - 1} C_U.$$

Unfortunately the value of C_A is a fraction of C_U : obtaining the necessary accuracy requires care in the layout.

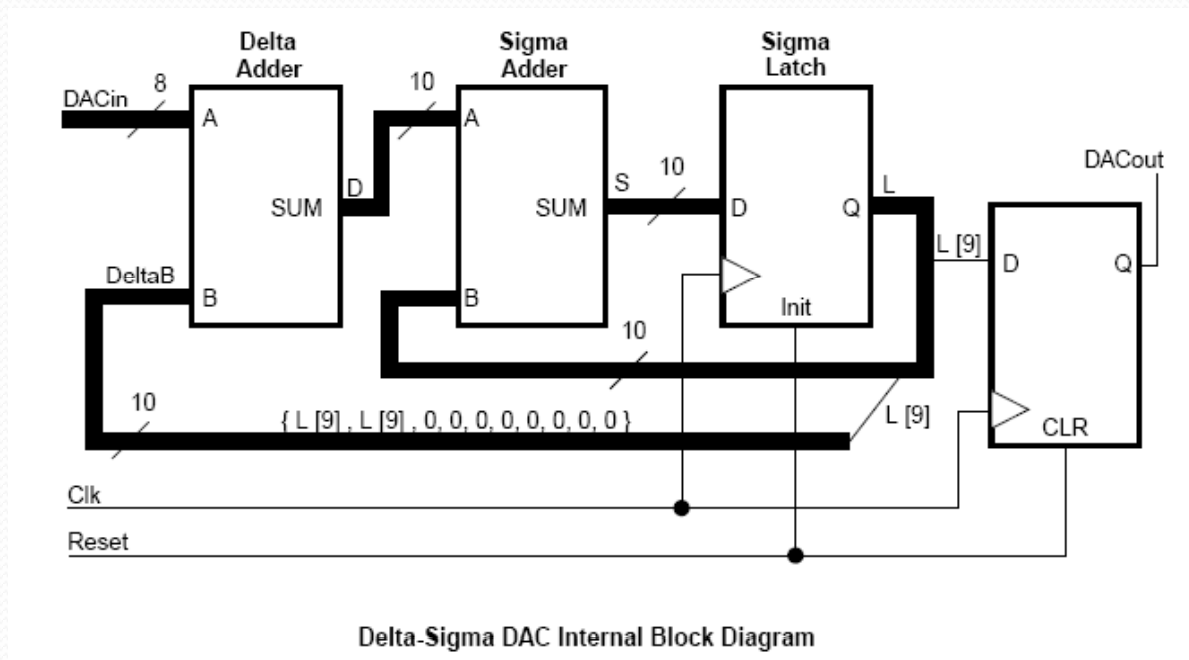
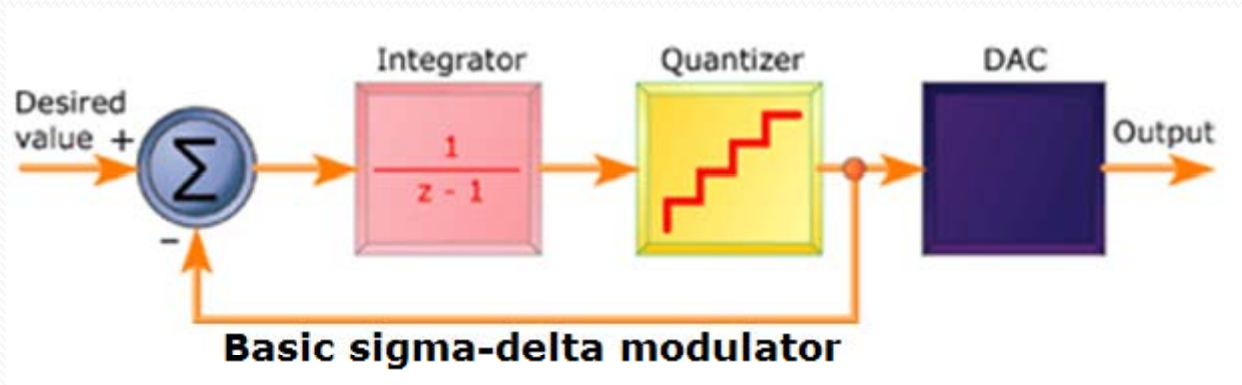
Reset phase



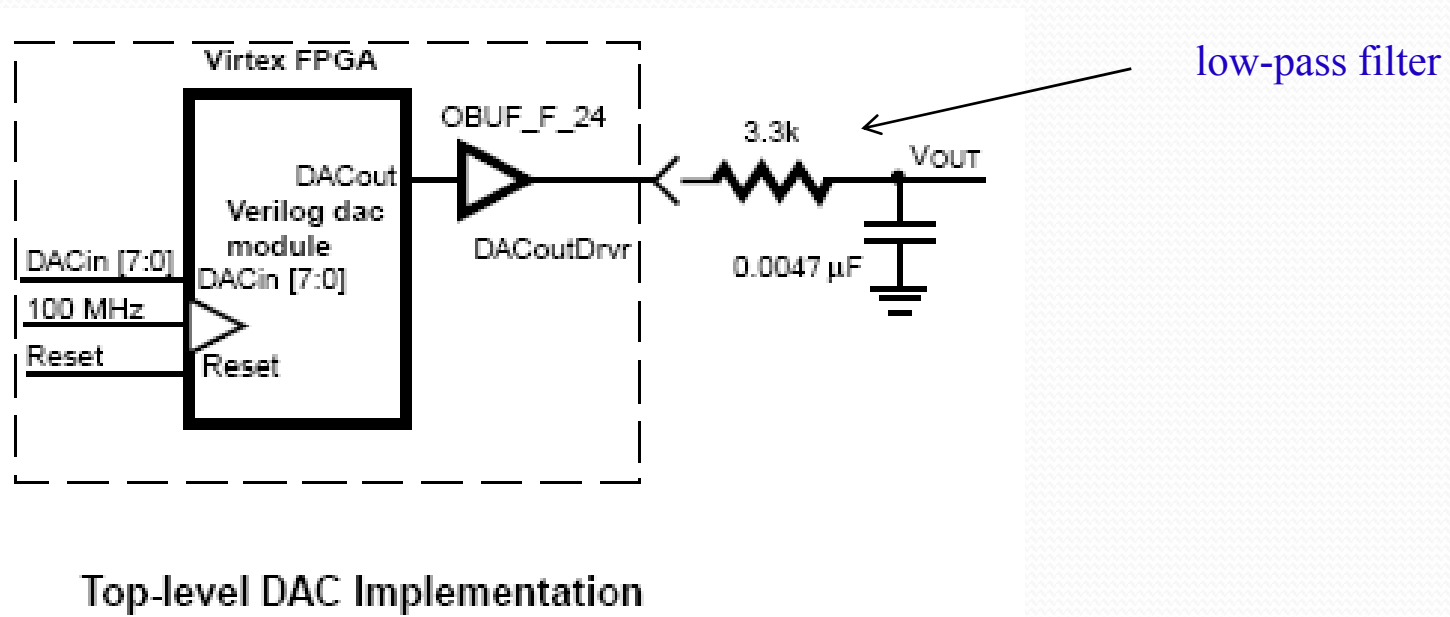
(b) The use of an attenuator in the middle of the array reduces the capacitance spread.

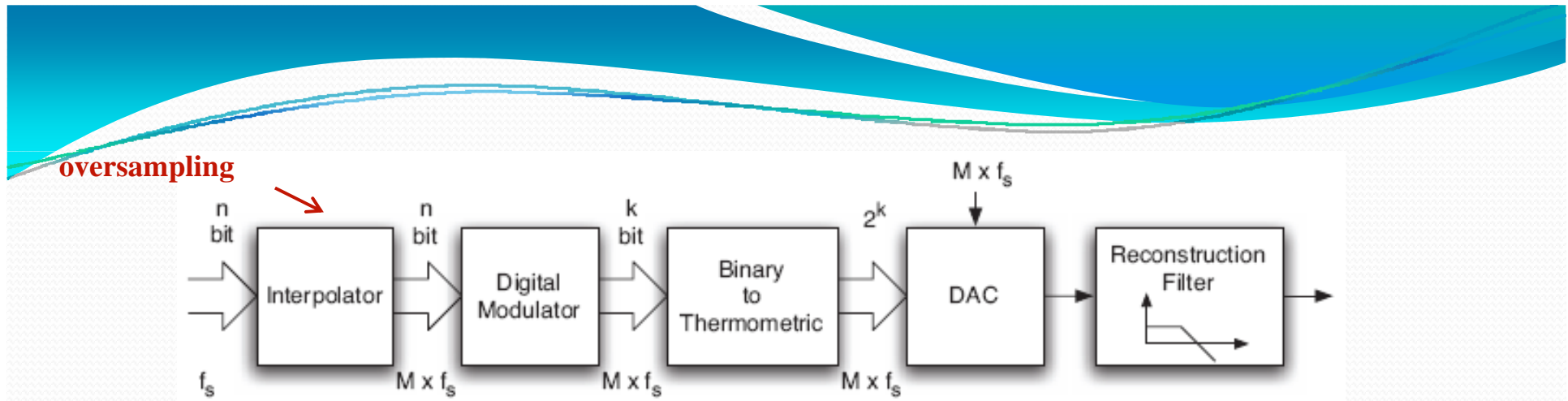
Oversampling (Σ - Δ) DAC

- basically, a Σ - Δ ADC in reverse!
- principle of operation of oversampled DAC is very similar to oversampled ADC (resolution increases with sampling frequency)
- improves SNR using noise shaping
- processing is done in the digital domain to generate a digital string at one-bit DAC output. The average duty-cycle of the output string is proportional to the value of the binary input
- the analog signal is created by passing the pulse string through an analog low-pass filter



- Delta Adder calculates the difference between input and current 1-bit DAC output (represented as a binary number)

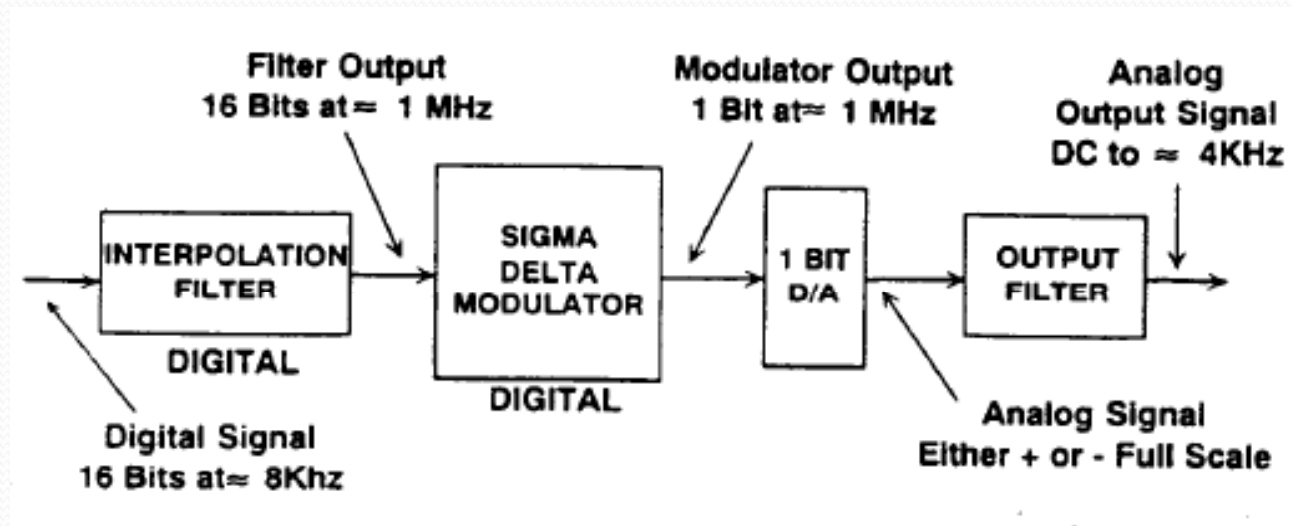




Block diagram of an oversampling DAC.

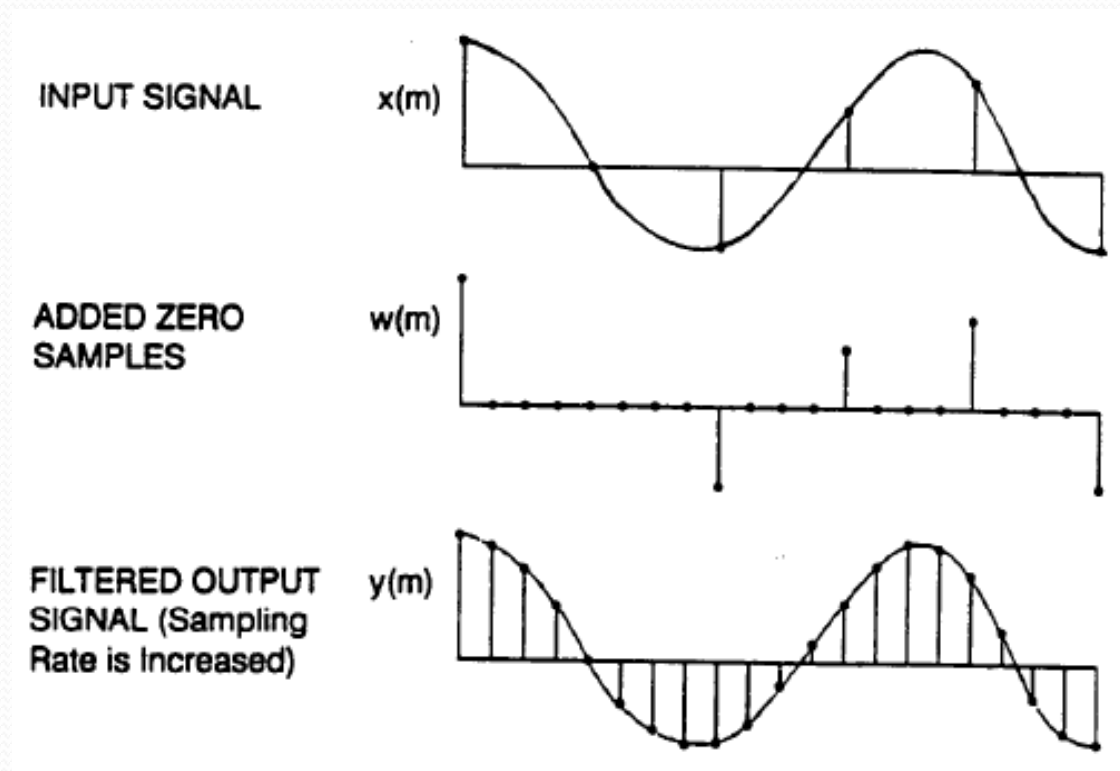
- interpolator increases the data-rate used for storing or transmitting digital signals to a much higher level: $2f_B \cdot \text{OSR}$ (reduce quantization noise) – commonly $2f_B$ close to Nyquist limit
- digital modulator reduces the number of bits for a possible thermometric representation
- the thermometric code is then used to control the low resolution DAC which precedes the reconstruction filter.


Ex:




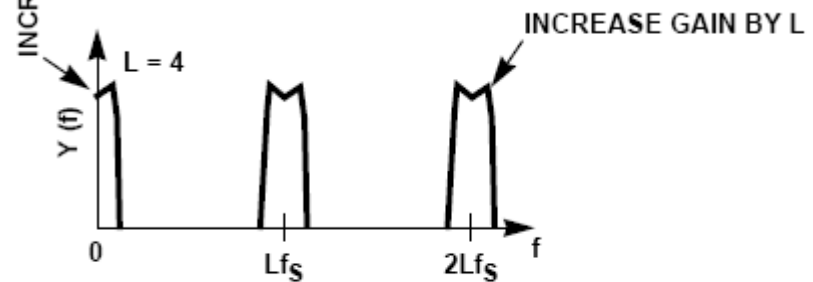
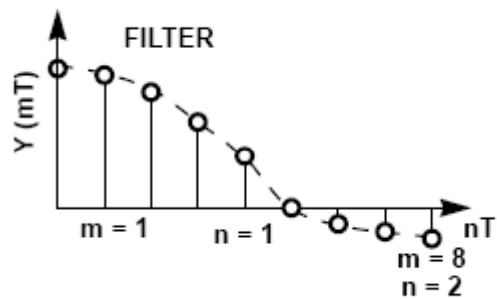
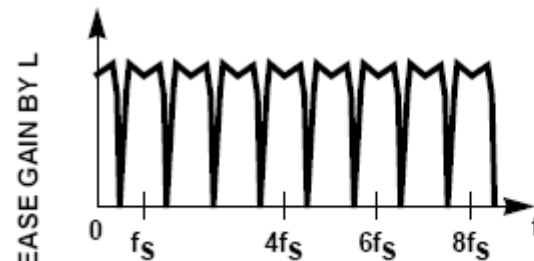
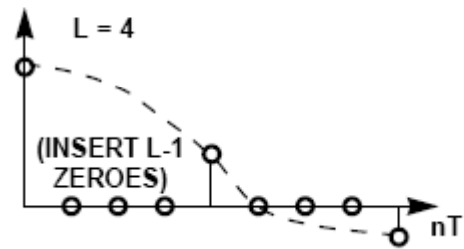
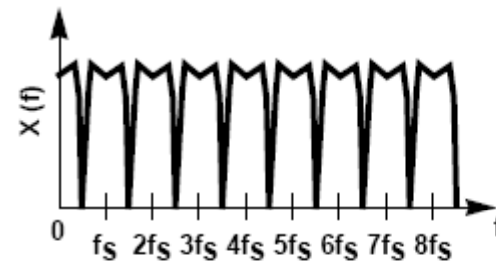
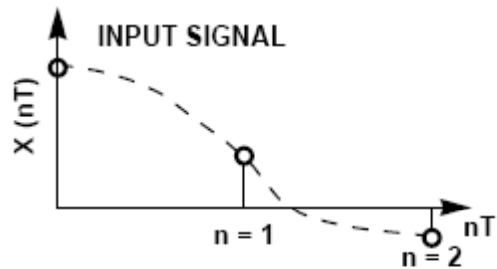
- **interpolation (upsampling)**

ex: interpolation by a factor $L = 4$

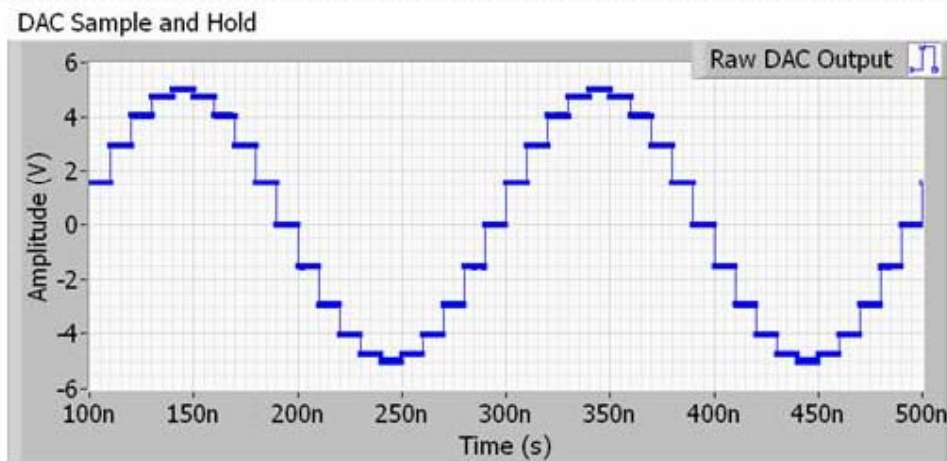


- 
- the interpolation factor (L) is normally large because a $\Sigma\Delta$ DAC aims at a significant reduction of the number of bit (at the limit it should go down to 1-bit)
 - oversampling is common in digital audio CD players, where the basic update rate of the data from the CD is 44.1 KSPS
 - early CD players used traditional binary DACs and inserted "zeros" into the parallel data, thereby increasing the effective update rate to 4-times, 8-times, or 16-times the fundamental throughput rate
 - the $4\times$, $8\times$, or $16\times$ data stream is passed through a digital interpolation filter which generates the extra data points.

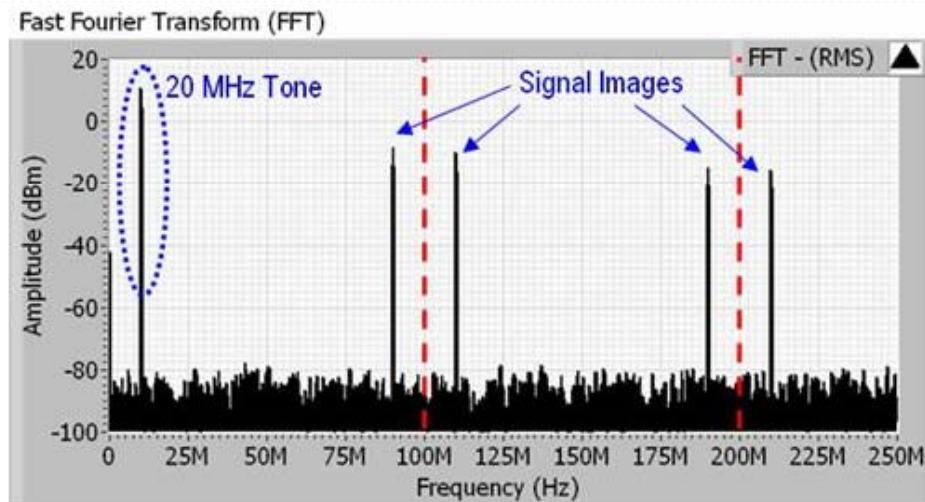
- 
- first step: to stuff $(L-1)$ zero-valued samples between each valid input sample, expanding the sampling rate by L , and causing the original signal spectrum to be repeated $L-1$ times.
 - to perform the actual interpolation, the zero-valued input samples must be converted to approximations of signal samples \Rightarrow equivalent to preserving the original signal spectrum.
 - effectively, the zero-stuffed input stream is filtered by a low-pass filter with a pass band at the original spectrum location. \Rightarrow this filters out all of the repeated spectra.



BLOCK DIAGRAM AND SPECTRAL REPRESENTATION OF THE INTERPOLATION PROCESS

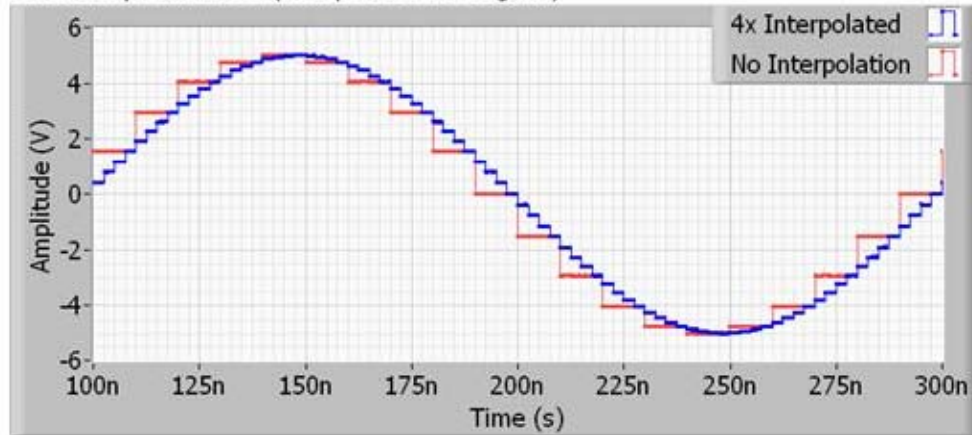


DAC Sample-and-Hold Output



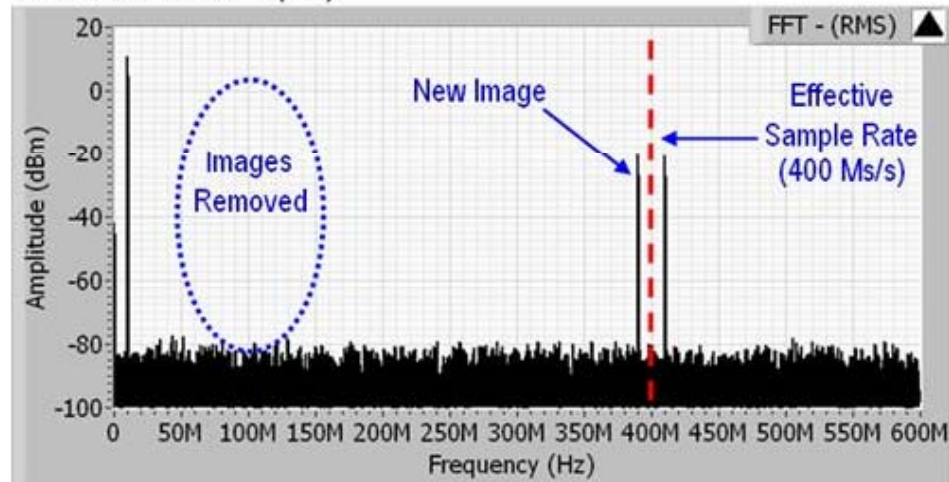
Spectral Images of a 20 MHz Sine Wave

DAC Sample and Hold (Interpolated vs. Original)



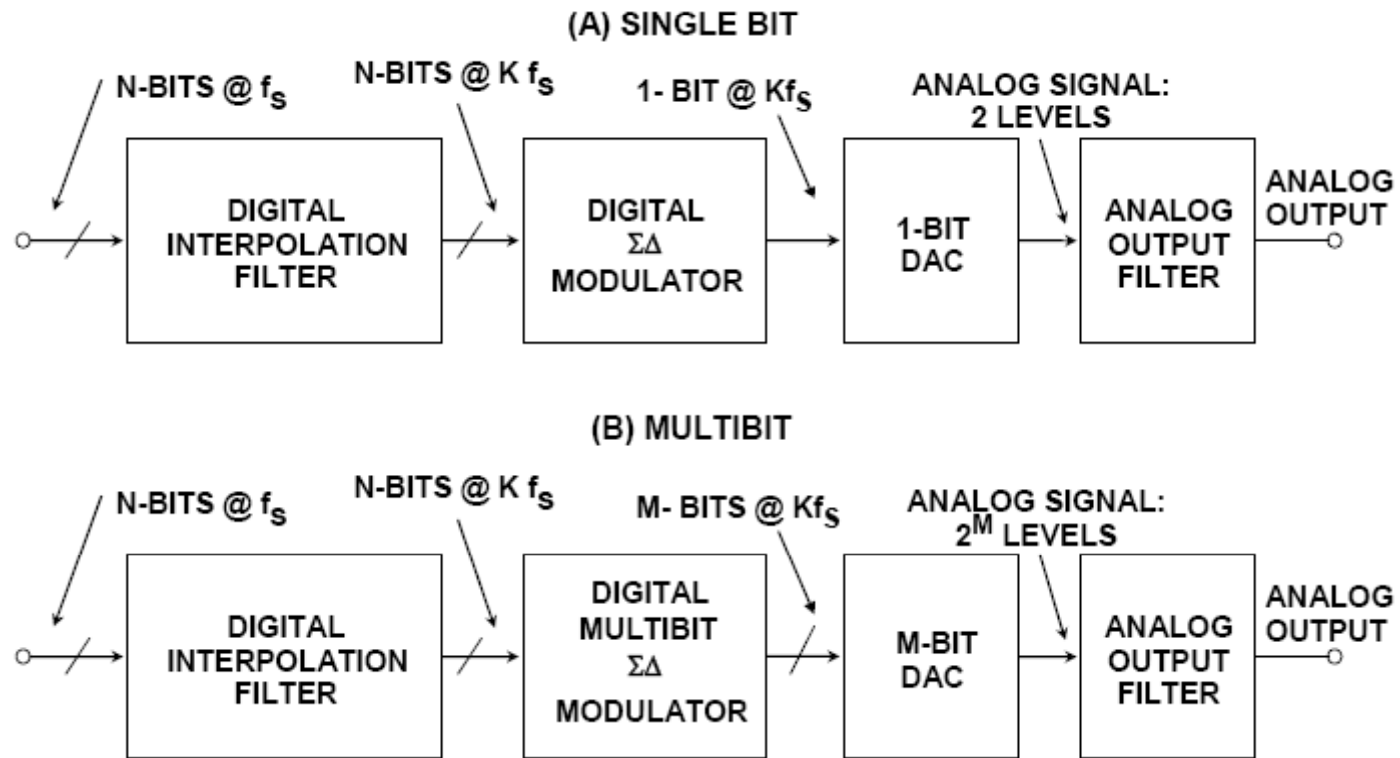
Interpolated DAC Output

Fast Fourier Transform (FFT)



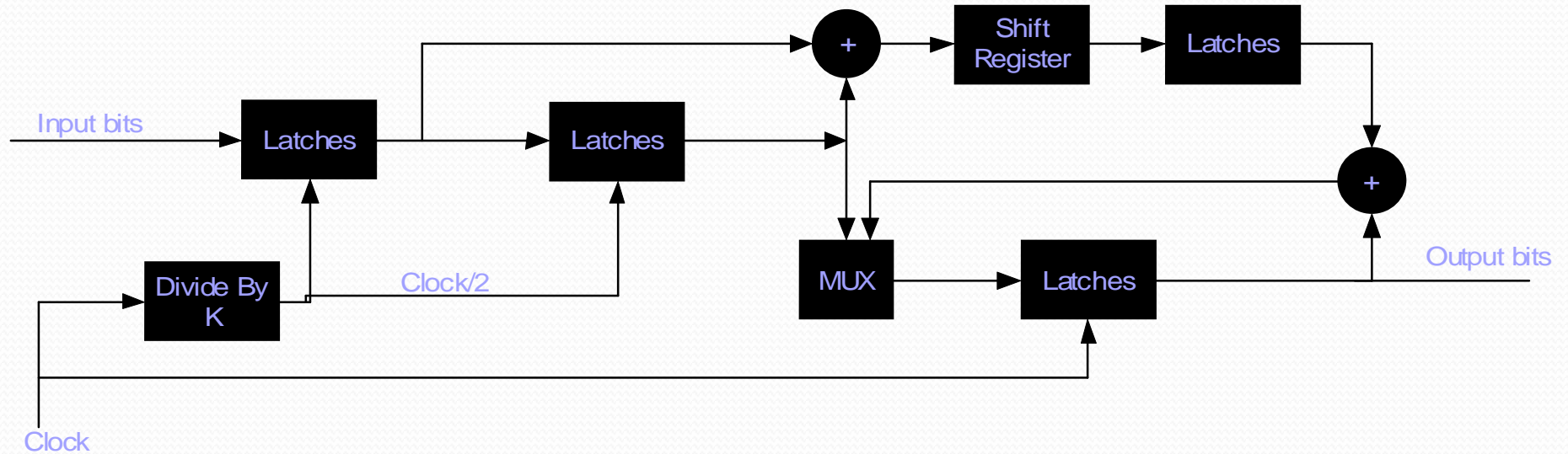
Spectral Images of a 20 MHz Sine Wave with 4X Interpolation

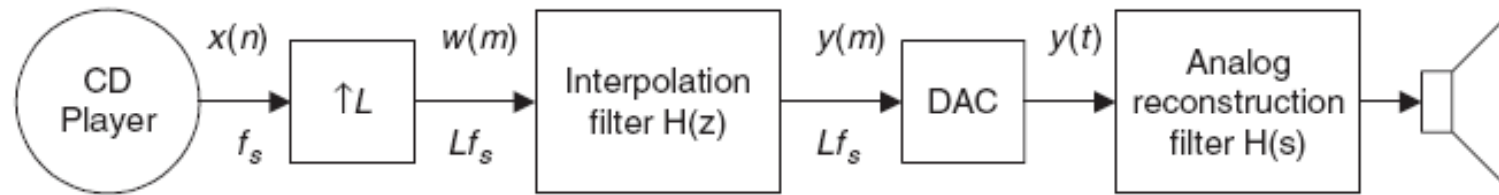
Σ - Δ DAC: fully digital



Sigma-Delta DACs


Interpolation filter block diagram

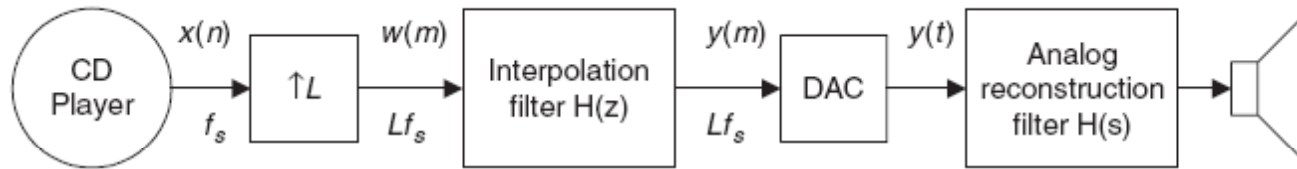




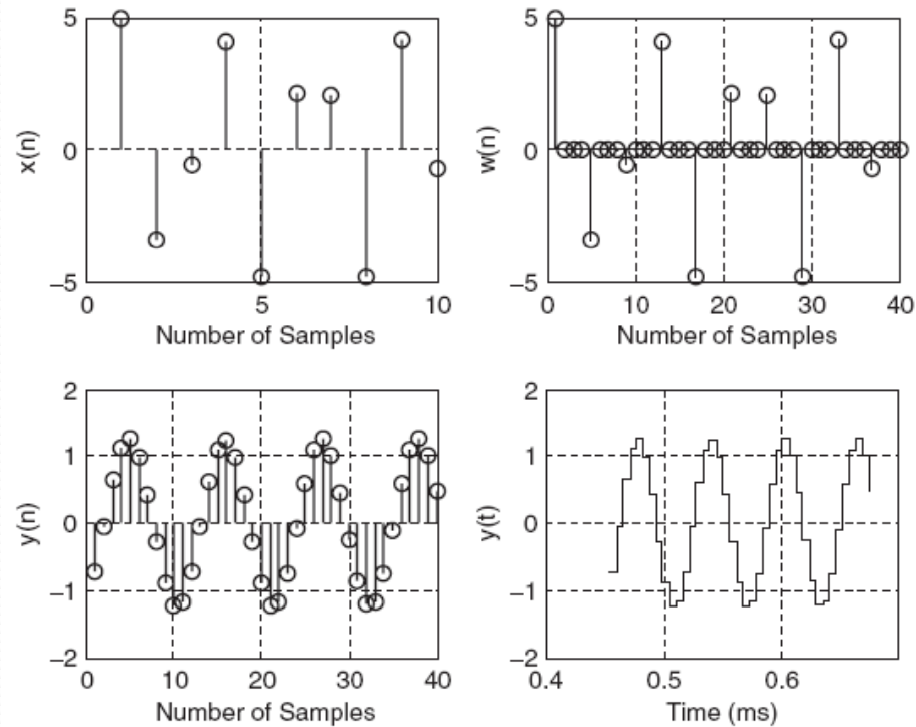
Sample rate conversion in the CD audio player system.

- each raw digital sample recorded on the CD audio system contains 16 bits and is sampled at the rate of 44.1 kHz.
- audio signal has a bandwidth of 22.05 kHz
- without upsampling and application of a digital interpolation filter
 \Rightarrow reconstruction filter (also called a smooth filter or anti-image filter) to remove all image frequencies beyond the Nyquist frequency of 22.05 kHz.
 \Rightarrow high-order filter required (complex)

- 
- **after digital interpolation**, the audio band is kept the same, while the sampling frequency is increased fourfold ($L = 4$), that is, $44.1 \times 4 = 176.4$ kHz
 - Since the audio band of 22.05 kHz is now relatively low compared with the new folding frequency ($176.4/2 = 88.2$ kHz), the use of a simple first-order or second-order analog anti-image filter may be sufficient.



Sample rate conversion in the CD audio player system.



Plots of signals at each stage according to the block diagram